

19.22 This problem asks that we treat a porous material as a composite wherein one of the phases is a pore phase, and that we estimate upper and lower limits for the room-temperature thermal conductivity of an aluminum oxide material having a 0.25 volume fraction of pores. The upper limit of  $k$  ( $k_{\text{upper}}$ ) may be determined using Equation 16.1 with thermal conductivity substituted for the elastic modulus,  $E$ . From Table 19.1, the value of  $k$  for  $\text{Al}_2\text{O}_3$  is 39 W/m-K, while for still air in the pore phase,  $k = 0.02$  W/m-K (Section 19.4). Thus

$$\begin{aligned} k_{\text{upper}} &= V_{\text{p}} k_{\text{air}} + V_{\text{Al}_2\text{O}_3} k_{\text{Al}_2\text{O}_3} \\ &= (0.25)(0.02 \text{ W/m-K}) + (0.75)(39 \text{ W/m-K}) = 29.3 \text{ W/m-K} \end{aligned}$$

For the lower limit we employ a modification of Equation 16.2 as

$$\begin{aligned} k_{\text{lower}} &= \frac{k_{\text{air}} k_{\text{Al}_2\text{O}_3}}{V_{\text{p}} k_{\text{Al}_2\text{O}_3} + V_{\text{Al}_2\text{O}_3} k_{\text{air}}} \\ &= \frac{(0.02 \text{ W/m-K})(39 \text{ W/m-K})}{(0.25)(39 \text{ W/m-K}) + (0.75)(0.02 \text{ W/m-K})} = 0.080 \text{ W/m-K} \end{aligned}$$