

19.13 This problem asks that we calculate the values of c_v for aluminum and iron at room temperature using Equation 19.10, the data in Table 19.1, given that $\alpha_v = 3\alpha_p$, and also values of the compressibility. From Equation 19.10

$$c_v = c_p - \frac{\alpha_v^2 v_0 T}{\beta}$$

And, from Table 19.1 and the problem statement

$$c_p(\text{Al}) = 900 \text{ J/kg-K}$$

$$c_p(\text{Fe}) = 448 \text{ J/kg-K}$$

$$\alpha_v(\text{Al}) = (3)[23.6 \times 10^{-6} (\text{°C})^{-1}] = 7.08 \times 10^{-5} (\text{°C})^{-1}$$

$$\alpha_v(\text{Fe}) = (3)[11.8 \times 10^{-6} (\text{°C})^{-1}] = 3.54 \times 10^{-5} (\text{°C})^{-1}$$

$$\beta(\text{Al}) = 1.77 \times 10^{-11} (\text{Pa})^{-1}$$

$$\beta(\text{Fe}) = 2.65 \times 10^{-12} (\text{Pa})^{-1}$$

The specific volume is just the reciprocal of the density; thus, in units of m^3/kg

$$v_0(\text{Al}) = \frac{1}{\rho} = \left(\frac{1}{2.71 \text{ g/cm}^3} \right) \left(\frac{1000 \text{ g}}{\text{kg}} \right) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^3 = 3.69 \times 10^{-4} \text{ m}^3/\text{kg}$$

$$v_0(\text{Fe}) = \left(\frac{1}{7.87 \text{ g/cm}^3} \right) \left(\frac{1000 \text{ g}}{\text{kg}} \right) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^3 = 1.27 \times 10^{-4} \text{ m}^3/\text{kg}$$

Therefore, for aluminum

$$\begin{aligned} c_v(\text{Al}) &= c_p(\text{Al}) - \frac{\alpha_v^2(\text{Al}) v_0(\text{Al}) T}{\beta(\text{Al})} \\ &= 900 \text{ J/kg-K} - \frac{[7.08 \times 10^{-5} (\text{°C})^{-1}]^2 (3.69 \times 10^{-4} \text{ m}^3/\text{kg})(293 \text{ K})}{1.77 \times 10^{-11} (\text{N/m}^2)^{-1}} \\ &= 869 \text{ J/kg-K} \end{aligned}$$

And, also for iron