19.12 (a) In this portion of the problem we are asked to determine the density of gold at 800°C on the basis of thermal expansion considerations. The basis for this determination will be 1 cm^3 of material at 20°C; this volume of gold has a mass of 19.320 g, which mass is assumed to remain constant upon heating to the 800°C. Let us first compute the volume expansion of this cubic centimeter of copper as it is heated to 800°C. According to Equation 19.4 volume expansion is equal to

$$\frac{\Delta V}{V_0} = \alpha_v \Delta T$$

where α_v, the volume coefficient of thermal expansion, as stipulated in the problem statement, is equal to $3\alpha_l$. The value of α_l given in Table 19.1 for gold is 14.2 x 10^{-6} $(°C)^{-1}$. Therefore, the volume of this specimen of Au at 800°C (V) is equal to

$$V = V_0 + \Delta V = V_0 + V_0 \alpha_v \Delta T = V_0 (1 + \alpha_v \Delta T)$$

$$= V_0 (1 + 3\alpha_l \Delta T) = V_0 \left[1 + 3\alpha_l (T_f - T_0)\right]$$

$$= (1 \text{ cm}^3) \left\{1 + (3) \left[14.2 \times 10^{-6} \, (°\text{C})^{-1}\right] (800°\text{C} - 20°\text{C})\right\}$$

$$= 1.03323 \text{ cm}^3$$

Thus, the density is just the 19.320 g divided by this new volume—i.e.,

$$\rho = \frac{19.320 \text{ g}}{1.03323 \text{ cm}^3} = 18.699 \text{ g/cm}^3$$

(b) Now we are asked to compute the density at 800°C taking into consideration the creation of vacancies which will further lower the density. To begin, this determination requires that we calculate the number of vacancies using Equation 4.1. But it first becomes necessary to compute the number of Au atoms per cubic centimeter (N_{Au}) at 800°C using Equation 4.2. Thus,

$$N_{Au} = \frac{N_A \, \rho_{Au}}{A_{Au}}$$

$$= \frac{(6.023 \text{ x } 10^{23} \text{ atoms/mol})(18.699 \text{ g/cm}^3)}{196.97 \text{ g/mol}}$$