NAME (underline family name):

STUDENT NUMBER: **SIGNATURE:**

FACULTY OF ENGINEERING
FINAL EXAMINATION
MATH 265
ADVANCED CALCULUS

Examiner: G. Schmidt **Date:** Friday, April 21, 2004
Associate Examiner: J. Loveys **Time:** 9:00 AM - 12:00 AM

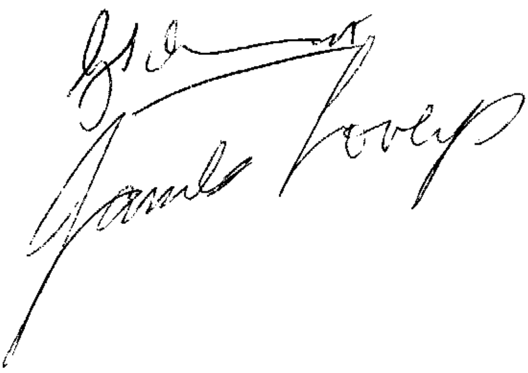
Instructions

1. Write your name and student number on this examination script.
2. All your answers must be given within this examination booklet. You may use the blank pages for rough work. You can also request extra paper for rough work, not to be handed in.
3. No books, calculators or notes allowed.
4. Answer all questions providing *full justification* for your answers.
5. Your answers may contain expressions that cannot be computed without a calculator.
6. Circle your answers where confusion could arise.
7. This examination booklet consists of this cover and 8 pages of questions.

GOOD LUCK!

Score Table

Problem	Points	Out of
1.		10
2.		10
3.		10
4.		10
5.		10
6.		10
7.		10
8.		10
Total:		80



1. (2 parts, 10 marks)

1(a) (8 marks) Verify that $y(t) = \cos 2t + \int_0^t \frac{1}{2} \sin 2(t-r)g(r)\,dr$ satisfies the differential equation $y''(t) + 4y(t) = g(t)$.

1(b) (2 marks) What are the initial values $y(0)$ and $y'(0)$?

2. (2 parts, 10 marks) Consider the pair of equations

$$zw + x^2z^3 + y^2w^3 + 1 = 0, \qquad x^2 + y^2 + xyzw - 3 = 0$$

2(a) (3 marks) Verify that one can solve the above equations to obtain functions $z(x, y)$ and $w(x, y)$ satisfying $z(1, -1) = 1$ and $w(1, -1) = -1$.

2(b) (7 marks) Evaluate the Jacobian determinant $\dfrac{\partial(z, w)}{\partial(x, y)}$ at $(x, y) = (1, -1)$.

3. (10 marks) Find the coordinates of the centroid of that part of the planar surface $3x + 2y - z = 3$ for which $1 \leq x - 2y \leq 3$ and $0 \leq 2x + y \leq 4$.

4. (3 parts, 10 marks) Consider the vector field

$$\mathbf{F}(x,y,z) = (y^2\cos x + z^3)\mathbf{i} + (2y\sin x - 4)\mathbf{j} + (3xz^2 + 2)\mathbf{k}.$$

4(a) (3 marks) Verify that $\mathbf{F}$ is conservative.

4(b) (4 marks) Find a potential function for $\mathbf{F}$.

4(c) (3 marks) Find the work done in moving an object in this field from $(0, 1, -1)$ to $(\pi/2, -1, 2)$.

5. (10 marks) Evaluate the line integral $\oint_C (\arctan x + y^2)\,dx + (e^y - x^2)\,dy$, where C is the boundary of the region determined by $1 \le x^2 + y^2 \le 9$ and $0 \le y$, oriented counterclockwise.

6. (10 marks) Let D be the region between $x^2+y^2+z=1$ and $x^2+y^2-z=1$ and S be the bounding surface. Evaluate the outward flux over S of the field

$$\mathbf{F}(x,y,z) = \mathbf{r} + \frac{\mathbf{r}}{|\mathbf{r}|^3} \quad \text{with } \mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}.$$

7. (10 marks) Verify Stokes' theorem (by explicitly evaluating both the surface and the line integrals) for the surface $z = 4 - x^2 - y^2$ with $z > 0$ and the vector field $\mathbf{F} = 2z\mathbf{i} + x\mathbf{j} + y^2\mathbf{k}$.

8. (2 parts, 10 marks)

8(a) (3 marks) Verify that for $\mathbf{F} = \mathbf{i} + z\mathbf{j}$ one has $\mathbf{F} \cdot (\nabla \times \mathbf{F}) \neq 0$.

8(b) (7 marks) Verify that if $\mathbf{F} = \phi \nabla \psi$, where ϕ and ψ are smooth functions, then $\mathbf{F} \cdot (\nabla \times \mathbf{F}) \equiv 0$.