**Written Assignment #4 (Due on Nov. 29, 2007)**

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This the assignment is on PDE and Fourier series. Please copy the question by hand; write out a full solution, and hand in the section of class that you registered.

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(1) Find the steady state solution for the heat equation:

$$u_t - 2u_{xx} = x^2 + 1, \quad (0 < x < 1, t > 0)$$

with the BC's:

$$u(0,t) + u'(0,t) = 1, \quad u(1,t) = 0, \quad (t \geq 0).$$

(2) (a) Find fourier series for the function

$$f(x) = \mid \sin x \mid, \quad \mid x \mid < \pi.$$

(b) Find Fourier cosine and Fourier sine series for the following function and sketch the Fourier series and compare function to its Fourier series:

$$f_0(x) = x, \quad 0 < x < \pi.$$

(c) Compute the Fourier cosine series for the function $g(x) = x - x^2/2$ define on $0 < x < 1$, and show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{6}\pi^2.$$

(3) We consider the IBVP of heat conduction of a rod with finite length:

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2},$$

with the periodic B.C's:

$$u(-\pi, t) = u(\pi, t), \quad u'(-\pi, t) = u'(\pi, t), \quad (t > 0)$$

and I.C.

$$u(x, 0) = 2x + |\sin x|, \quad (-\pi < x < \pi).$$

(4) We consider the IBVP of vibration of a string with finite length:

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2},$$

with B.C's:

$$u(-1,t) = u(1,t) = 0, \quad (t > 0)$$

and I.C.

$$\begin{cases} u(x,0) = (1-x^2) \\ \dfrac{\partial u}{\partial t}(x,0) = 0. \end{cases} \qquad (-1 < x < 1)$$

(5) Find solution $u(x,y)$ if

$$\nabla^2 u(x,y) = 0, \quad (0 < x, y < \pi),\ t > 0),$$

and

$$u(0,y) = u(\pi,y) = 0, \quad (0 < y < \pi),$$

$$\begin{aligned} &u_y(x,0) = x(x-\pi); \\ &u_y(x,\pi) = (\cos^2 x + 3\sin x), \\ &\quad (0 < x < \pi). \end{aligned}$$