Simon M. Foucher **Assignment 3** **MATH133, Fall 2006**
due 10/8/06 at 11:00 PM.

1.(2 pts) In each of the following problems, decide whether **u** is in the span of **v** and **w** or not.

? 1. $\mathbf{u} = \begin{bmatrix} 3 \\ -4 \\ 16 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -4 \\ -4 \\ 5 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 4 \\ -5 \\ -4 \end{bmatrix}$

? 2. $\mathbf{u} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 6 \\ 5 \\ -1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} -3 \\ -7 \\ -4 \end{bmatrix}$

? 3. $\mathbf{u} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -5 \\ -2 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

? 4. $\mathbf{u} = \begin{bmatrix} 9 \\ -2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$

? 5. $\mathbf{u} = \begin{bmatrix} 10 \\ -4 \\ 10 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ -1 \\ -4 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 3 \\ -4 \\ -3 \end{bmatrix}$

2.(2 pts) Check the following statements for validity:

? 1. $\mathbb{R}^3 = span\{ \begin{bmatrix} 5 \\ -9 \\ 9 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -4 \end{bmatrix} \}$

? 2. $\mathbb{R}^2 = span\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \end{bmatrix} \}$

? 3. $\mathbb{R}^3 = span\{ \begin{bmatrix} 16 \\ 14 \\ -6 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 18 \\ 13 \\ -5 \end{bmatrix}, \begin{bmatrix} -3 \\ -4 \\ 2 \end{bmatrix} \}$

? 4. $\mathbb{R}^3 = span\{ \begin{bmatrix} -2 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} -7 \\ 7 \\ 1 \end{bmatrix} \}$

? 5. $\mathbb{R}^2 = span\{ \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 9 \\ -7 \end{bmatrix} \}$

3.(2 pts) In each of the following problems, write the first vector as a linear combination of the remaining vectors. If this is impossible, set all coefficients to NA. If there is more than one solution, enter one of them.

1. $\begin{bmatrix} 4 \\ 8 \end{bmatrix} = \_\_ \begin{bmatrix} 4 \\ 8 \end{bmatrix} + \_\_ \begin{bmatrix} -9 \\ -2 \end{bmatrix}$

2. $\begin{bmatrix} 16 \\ 1 \end{bmatrix} = \_\_ \begin{bmatrix} 4 \\ 1 \end{bmatrix} + \_\_ \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

3. $\begin{bmatrix} -4 \\ -3 \end{bmatrix} = \_\_ \begin{bmatrix} 10 \\ -6 \end{bmatrix} + \_\_ \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

4. $\begin{bmatrix} 9 \\ -3 \\ 0 \end{bmatrix} = \_\_ \begin{bmatrix} -5 \\ 1 \\ 3 \end{bmatrix} + \_\_ \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$

5. $\begin{bmatrix} 9 \\ -3 \\ 0 \end{bmatrix} = \_\_ \begin{bmatrix} 1 \\ -5 \\ 1 \end{bmatrix} + \_\_ \begin{bmatrix} -5 \\ 1 \\ 3 \end{bmatrix} + \_\_ \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$

4.(2 pts) In each of the following problems, decide whether the given sets of vectors are linearly dependent or linearly independent.

? 1. $\left\{ \begin{bmatrix} 1 \\ 2 \\ -5 \\ -5 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 4 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ -4 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 0 \\ -4 \end{bmatrix} \right\}$

? 2. $\left\{ \begin{bmatrix} -1 \\ -9 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 7 \\ 0 \\ 8 \end{bmatrix} \right\}$

? 3. $\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ -5 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ -3 \\ 3 \end{bmatrix} \right\}$

? 4. $\left\{ \begin{bmatrix} 6 \\ 0 \\ -6 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -6 \\ 0 \\ 1 \end{bmatrix} \right\}$

? 5. $\left\{ \begin{bmatrix} 1 \\ -5 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ -13 \\ -1 \\ 11 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 5 \\ -1 \end{bmatrix} \right\}$

5.(2 pts) The solution set of the homogeneous linear system

$$\begin{array}{rrrrr} 1x_1 & +1x_2 & +7x_3 & +3x_4 & -5x_5 = 0 \\ 1x_1 & +2x_2 & +10x_3 & +4x_4 & -9x_5 = 0 \\ -3x_1 & +0x_2 & -12x_3 & -5x_4 & +0x_5 = 0 \\ 2x_1 & +1x_2 & +11x_3 & +5x_4 & -6x_5 = 0 \\ -3x_1 & -4x_2 & -24x_3 & -10x_4 & +19x_5 = 0 \end{array}$$

is spanned by the the vectors
u = [____,____,____,____,____]
v = [____,____,____,____,____]

6.(1 pt) (a) Let **u,v,w** be linearly independent vectors. A non-trivial dependence relation satisfied by the vectors
8**u** −96 **v** , 4 **v + w** , 3**w + u**
is
____(8**u** −96 **v**) +____(4 **v + w**) + ____(3**w + u**) = **0** .
(b) **A complete solution to this part is to be handed in with Written Assignment 2.**
If **u,v,w** are linearly independent vectors prove that
8**u** +96 **v** , 4 **v + w** , 3**w + u**
are also linearly independent.