**ECSE420 - Parallel Computing**

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**Multi Thread Performance Analysis of a Brute-Force Sudoku Solver**

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## The game

Sudoku is a popular puzzle game similar to crosswords, but using numbers. The Sudoku grid is a square, divided in subsquares containing as many elements as the length of the row of the main square. Which such a partition, every element in the Sudoku board has 3 constraints: row, column and sub square. The constraint stipulates that a number, ranging from 1 to the size of a subsquare, an only be places once in a row, a column and a sub square.

 There are many strategies one can use to solve Sudoku puzzles, and solutions might not be unique, or might not even exist! The difficulty rating of a board is relative to the number of indices provided. A standard Sudoku board contains 81 places, and is subdivided into 9 subsquares each containing 9 elements.

## Approach

 When solved by a computer, one of the most complete approaches to solving a Sudoku is using brute force. This provides 2 advantages over algorithmic solvers: first, it can confirm if there are no solutions, and second, if there are many solutions, the computer can easily output a complete list of them. In this work we have developed a multi-threaded brute force Sudoku solver. To standardize the approach, the same board has been used throughout the development. The puzzle is rated as “Extremely difficult” and has been taken from:

http://en.wikipedia.org/wiki/Algorithmics\_of\_sudoku



**Figure 1: the Sudoku puzzle used for gathering the performance data.**

 The experiment will consist in repeatedly solving this puzzle using the same algorithm with a varying number of threads to observe the variation in performance obtained.

## Algorithm

 The algorithm uses recursive calls on a test-and-backtrack approach. All the cells of the board will be visited.

1. Upon arriving, if the cell is not empty, it is skipped. Otherwise, a temporary value is assigned from 1 to 9. In figure 2, we can observe that the first cell (0,0) got skipped and that the value of ‘1’ has been assigned to the next cell (0,1)



**Figure2: Testing the legality of a ‘1’ at the position (0,1**

1. Afterwards, a test is performed to see if the attempted value is allowed there. By scanning the 3 dimensions of neighbors, the new value is compared to all the elements contained in its row, its column and its sub square. If the same element is encountered, the next value is tested and the algorithm repeats the test. If all the values have been exhausted, the algorithm backtracks to the previous cell.
2. This process is repeated until the program finds an element that could potentially exist at that location



**Figure 3: ‘3’ is the first ‘legal’ value we can leave at position (0,1) before moving on to the next position**

1. Once this condition is satisfied, the function is recursively called to the next location. To optimize the software, when the function is called a second time, the first element it looks at is the previous element incremented by 1. For example here, there since we just guessed ‘3 to be at location (0,1), the algorithm will guess ‘4’ the next location
2. Once the depth of the recursion reaches 81, we know that all the cells have been visited, so if no solution was found, we backtrack one cell and keep trying all the possibilities on that one by recalling the recursion.
3. This keeps happening until all the possibilities have been tried out. If no solution was bound in the mean time, an error message is printed on the board (for the purpose of this experiment, we know that there is a solution so it is always found sooner or later)

## Optimizing brute force

 By definition, a brute force algorithm is the least optimal solution to a problem, but guarantees results. The time required to solve the puzzle greatly depends on where the search is conducted, and which elements are visited first. It is even possible, although unlikely to take O(1) time if the first try is the right answer.

 It might have been tempting to try and optimize the solver by testing various starting positions and various start number, but this optimization would have only been valid for this particular puzzle. We felt that the best approach was to randomize the start location, as well as the start guess. In order to implement this, we used circular loops to scan the rows, columns and tested values. To keep track of how ‘deep’ we got in the algorithm, we incremented an index at every function call that made the function exit when it reached 81 (the maximum depth of the grid).

 This approach also gives us great versatility with regards to multithreading environment. By starting at a random location, the solving function can be called by many threads and generate good ‘far apart’ starting states for each thread. This added feature gives the user to freely add threads without having to modify anything in the algorithm for the new threads to pick up a work load.

## Multi-Threading and synchronization

 Since the challenge is to test a very large possibility space, the use of parallel computing can greatly aid. For this particular problem, we used many threads to run a subset of the problem. Java was chosen as a platform because of its ease of implementation of multithreading environments. The main class takes care of creating threads, and every thread takes on a portion of the puzzle. As soon as one of them solves the problem, it prints out the solution and the time it took to solve it.

 To handle synchronization, we used a global variable called ‘finished’. When the main function starts, if sets ‘finished’ to false. As the threads try to solve the puzzle, they look at this variable and exit if it is true, otherwise continue to try and solve. As soon as a thread solves the puzzle, it sets the variable to true, which forces all the other threads out of their solving function. Meanwhile, in the main function, after all the threads have been initialized, the program falls into a while(!finished) loop. As soon as the barrier is crossed, we know that a thread has solved the puzzle, so all the threads are destroyed.

## Experiment

 Since, in general, we do not have *a priori* information on the best starting location for a brute force scan; we felt the need to test various scenarios to make sure that the data collected was a good reflection of reality. Therefore, in order to gather a comparative idea of various times to solve, rather than the particular value for this puzzle, many runs of the same test were performed. We also tested for many different numbers of threads.

 The tests were conducted on an Inter Dual core processor @1.4GHz, managed by the Windows 7 platform.

## REsults

 First we ran some tests on a single thread to see how long it took on average to solve the puzzle. Here are the results we found.

**Figure 4: Individual results of a single threaded application solving the puzzle. On the x axis, we can see how long it took, and on the right, we can see the trial number.**

 These results justified the assumption made that the time it takes to fully solve a Sudoku puzzle on a brute force approach is greatly relative to the starting location. We can observe a few instances where it took 3-4 full seconds to solve, these represents where statistically we picked the worst possible starting point and the solution was one of the last ones we tested. As the other extreme, we can observe some instances where the puzzle was solved in a matter of milliseconds. These instances occur when the solution is one of the first ones to be found.

 Afterwards, we ran the application with a varying number of threads working on solving the puzzle, from 1 to 10, and got the following results, which are similar to the original results we got. For any given number of threads, 500 tests were performed to ensure that the data gathered was statistically significant. (The number of different starting positions is 9x9 with 9 possible starting values, giving 729 possible initial states for the system.)

**Figure 5: Combined results of all the tests performed with a varying number of threads. On the x axis, we can observe the time to solve, in ms.**

**Figure 6: Average time to solve when using a variation of number of threads. On the x axis we can see the number of threads used, and on the y axis we can see the average time to solve in ms.**

 These are interesting results, which require a few explanations. First, as expected by running the code on a dual core processor, we can see that there is a sharp decrease in the average time required to solve the puzzle when going from a single threaded application to using 2 threads. This can intuitively be explained because of the fact that 2 processors can easily handle 2 threads without too much overhead.

When adding a 3rd and 4th thread, we can see that the average time to solve increased almost as far up as the original time it took for a single threaded application. This is caused by the added overhead brought by managing the extra threads without bringing too many benefits.

Afterwards, we observe a gradual decrease in the time to solve as the number of processors keeps increasing, until it bottoms out at 100ms. This continuous decrease is most probably due to the random nature of the starting position and starting number. As we can observe in figure 4, when comparing the performance we got from different starting locations, the data is mostly gathered in very low solving times (<200ms), with a few high peaks (>1s), which drive the average up. Since the vast majority of the starting positions can yield a fast answer on a brute force approach, by adding threads starting at random locations, we increase the chance that at least one of them starts in that location. If this is the case, as soon as this thread reaches the solution, all the other ones will be stopped anyways.

Finally, as the number of threads keeps increasing, we can observe a slight increase in the average time to solve. This average shows a weak asymptote at around 100ms, with a slight positive slope, indicating that the added threads do induce some extra overhead.

# Appendix I: source code used

The code consists of a single java file and was developed using the IDE Eclipse, and has been inspired by an algorithm proposed by Bob Carpenter on the following web page: <http://www.colloquial.com/games/sudoku/java_sudoku.html>.

package sudokuSolver;

import java.util.Random;

// Class taking care of solving the Sudoku

public class solver extends Thread {

 static boolean finished;

 // This method is called when a thread starts running

 // It will call the recursive solver and output the data

 public void run() {

 int[][] board = setUpBoard(1); // Set up the board (option to select different puzzles)

 int i, j, value; // Start position (i,j) and start value will be randomly selected

 long start, end;

 Random generator = new Random(System.currentTimeMillis());

 //printBoard(board);

 board = setUpBoard(1);

 i = generator.nextInt(8);

 j = generator.nextInt(8);

 value = generator.nextInt(8)+1;

 start = System.currentTimeMillis();

 if (solve(i, j, board, 0, value)) ; // solves in place

 //printBoard(board);

 else

 System.out.println("No answers for this puzzle!");

 end = System.currentTimeMillis();

 if(!finished){

 finished = true;

 System.out.print((end - start)+"\n");

 }

 }

 public static void main(String[] args) {

 // 1 threads thread

 System.out.println("\n\n1\nTime (ms)");

 for(int k = 0; k < 400; k++){

 finished = false;

 Thread thread1 = new solver();

 thread1.start();

 while(!finished);

 thread1.stop();

 }

 // 2 threads thread

 System.out.println("\n\n2\nTime (ms)");

 for(int k = 0; k < 400; k++){

 finished = false;

 Thread thread1 = new solver();

 Thread thread2 = new solver();

 thread1.start();

 thread2.start();

 while(!finished);

 thread1.stop();

 thread1= null;

 thread2.stop();

 thread2 = null;

 }

 // 3 threads thread

 System.out.println("\n\n3\nTime (ms)");

 for(int k = 0; k < 400; k++){

 finished = false;

 Thread thread1 = new solver();

 Thread thread2 = new solver();

 Thread thread3 = new solver();

 thread1.start();

 thread2.start();

 thread3.start();

 while(!finished);

 thread1.stop();

 thread1= null;

 thread2.stop();

 thread2 = null;

 thread3.stop();

 thread3= null;

 }

 // 4 threads thread

 System.out.println("\n\n4\nTime (ms)");

 for(int k = 0; k < 400; k++){

 finished = false;

 Thread thread1 = new solver();

 Thread thread2 = new solver();

 Thread thread3 = new solver();

 Thread thread4 = new solver();

 thread1.start();

 thread2.start();

 thread3.start();

 thread4.start();

 while(!finished);

 thread1.stop();

 thread1= null;

 thread2.stop();

 thread2 = null;

 thread3.stop();

 thread3= null;

 thread4.stop();

 thread4 = null;

 }

 // 5 threads thread

 System.out.println("\n\n5\nTime (ms)");

 for(int k = 0; k < 400; k++){

 finished = false;

 Thread thread1 = new solver();

 Thread thread2 = new solver();

 Thread thread3 = new solver();

 Thread thread4 = new solver();

 Thread thread5 = new solver();

 thread1.start();

 thread2.start();

 thread3.start();

 thread4.start();

 thread5.start();

 while(!finished);

 thread1.stop();

 thread1= null;

 thread2.stop();

 thread2 = null;

 thread3.stop();

 thread3= null;

 thread4.stop();

 thread4 = null;

 thread5.stop();

 thread5 = null;

 }

 // 6 threads thread

 System.out.println("\n\n6\nTime (ms)");

 for(int k = 0; k < 400; k++){

 finished = false;

 Thread thread1 = new solver();

 Thread thread2 = new solver();

 Thread thread3 = new solver();

 Thread thread4 = new solver();

 Thread thread5 = new solver();

 Thread thread6 = new solver();

 thread1.start();

 thread2.start();

 thread3.start();

 thread4.start();

 thread5.start();

 thread6.start();

 while(!finished);

 thread1.stop();

 thread1= null;

 thread2.stop();

 thread2 = null;

 thread3.stop();

 thread3= null;

 thread4.stop();

 thread4 = null;

 thread5.stop();

 thread5 = null;

 thread6.stop();

 thread6 = null;

 }

 // 7 threads thread

 System.out.println("\n\n7\nTime (ms)");

 for(int k = 0; k < 400; k++){

 finished = false;

 Thread thread1 = new solver();

 Thread thread2 = new solver();

 Thread thread3 = new solver();

 Thread thread4 = new solver();

 Thread thread5 = new solver();

 Thread thread6 = new solver();

 Thread thread7 = new solver();

 thread1.start();

 thread2.start();

 thread3.start();

 thread4.start();

 thread5.start();

 thread6.start();

 thread7.start();

 while(!finished);

 thread1.stop();

 thread1= null;

 thread2.stop();

 thread2 = null;

 thread3.stop();

 thread3= null;

 thread4.stop();

 thread4 = null;

 thread5.stop();

 thread5 = null;

 thread6.stop();

 thread6 = null;

 thread7.stop();

 thread7 = null;

 }

 // 8 threads thread

 System.out.println("\n\n8\nTime (ms)");

 for(int k = 0; k < 400; k++){

 finished = false;

 Thread thread1 = new solver();

 Thread thread2 = new solver();

 Thread thread3 = new solver();

 Thread thread4 = new solver();

 Thread thread5 = new solver();

 Thread thread6 = new solver();

 Thread thread7 = new solver();

 Thread thread8 = new solver();

 thread1.start();

 thread2.start();

 thread3.start();

 thread4.start();

 thread5.start();

 thread6.start();

 thread7.start();

 thread8.start();

 while(!finished);

 thread1.stop();

 thread1= null;

 thread2.stop();

 thread2 = null;

 thread3.stop();

 thread3= null;

 thread4.stop();

 thread4 = null;

 thread5.stop();

 thread5 = null;

 thread6.stop();

 thread6 = null;

 thread7.stop();

 thread7 = null;

 thread8.stop();

 thread8 = null;

 }

 // 9threads thread

 System.out.println("\n\n9\nTime (ms)");

 for(int k = 0; k < 400; k++){

 finished = false;

 Thread thread1 = new solver();

 Thread thread2 = new solver();

 Thread thread3 = new solver();

 Thread thread4 = new solver();

 Thread thread5 = new solver();

 Thread thread6 = new solver();

 Thread thread7 = new solver();

 Thread thread8 = new solver();

 Thread thread9 = new solver();

 thread1.start();

 thread2.start();

 thread3.start();

 thread4.start();

 thread5.start();

 thread6.start();

 thread7.start();

 thread8.start();

 thread9.start();

 while(!finished);

 thread1.stop();

 thread1= null;

 thread2.stop();

 thread2 = null;

 thread3.stop();

 thread3= null;

 thread4.stop();

 thread4 = null;

 thread5.stop();

 thread5 = null;

 thread6.stop();

 thread6 = null;

 thread7.stop();

 thread7 = null;

 thread8.stop();

 thread8 = null;

 thread9.stop();

 thread9 = null;

 }

 // 10 threads thread

 System.out.println("\n\n10\nTime (ms)");

 for(int k = 0; k < 400; k++){

 finished = false;

 Thread thread1 = new solver();

 Thread thread2 = new solver();

 Thread thread3 = new solver();

 Thread thread4 = new solver();

 Thread thread5 = new solver();

 Thread thread6 = new solver();

 Thread thread7 = new solver();

 Thread thread8 = new solver();

 Thread thread9 = new solver();

 Thread thread10 = new solver();

 thread1.start();

 thread2.start();

 thread3.start();

 thread4.start();

 thread5.start();

 thread6.start();

 thread7.start();

 thread8.start();

 thread9.start();

 thread10.start();

 while(!finished);

 thread1.stop();

 thread1= null;

 thread2.stop();

 thread2 = null;

 thread3.stop();

 thread3= null;

 thread4.stop();

 thread4 = null;

 thread5.stop();

 thread5 = null;

 thread6.stop();

 thread6 = null;

 thread7.stop();

 thread7 = null;

 thread8.stop();

 thread8 = null;

 thread9.stop();

 thread9 = null;

 thread10.stop();

 thread10 = null;

 }

 }

 static int[][] setUpBoard(int puzzleChoise) {

 // Set up a new board, all formatted to 0

 int[][] newBoard = new int[9][9]; // default 0 vals

 int i,j, n=0;

 // Puzzles are entered in standard format

 int[] puzzle = {1,0,0,0,0,0,0,0,2,0,9,0,4,0,0,0,5,0,0,0,6,0,0,0,7,0,0,0,5,0,9,0,3,0,0,0,0,0,0,0,7,0,0,0,0,0,0,0,8,5,0,0,4,0,7,0,0,0,0,0,6,0,0,0,3,0,0,0,9,0,8,0,0,0,2,0,0,0,0,0,0};

 for(i = 0; i < 9; i++)

 for(j= 0; j < 9; j++)

 newBoard[i][j] = puzzle[n++];

 // printBoard(newBoard);

 return newBoard;

 }

 static boolean solve(int row, int col, int[][] board,int xTimes, int startV) {

 // Return true if we have reached the depth of the recursion

 if(xTimes == 81) return true;

 if(finished) return true;

 // Do a loop of rows and columns

 if (++col == 9){

 col = 0;

 if(++row == 9)

 row = 0;

 }

 if (board[row][col] != 0){ // skip filled cells

 return solve(row ,col, board, xTimes+1, startV);

 }

 // The code can only get here if the cell was 0

 for (int val = 1; val <= 9; ++val) {

 if(++startV == 10) startV = 1;

 // first check of the value is allowed here

 if (allowedHere(row,col,startV,board)) {

 board[row][col] = startV; // If allowed, record it and run recursively

 if(solve(row ,col, board, xTimes+1, startV))

 return true;

 }

 }

 board[row][col] = 0; // reset on backtrack

 return false;

 }

 static boolean allowedHere(int row, int col, int value, int[][] board) {

 int i;

 // scan 9 neighboring possibilities

 for(i = 0; i < 9; i++){

 // look at colums in this row

 if(board[row][i] == value)

 return false;

 // look at rows in this column

 if(board[i][col] == value)

 return false;

 // look at sub square

 if (board[row/3\*3+i%3][col/3\*3+i/3] == value)

 return false;

 }

 return true; // no violations, so it's legal

 }

 static void printBoard(int[][] boardToPrint) {

 int i, j;

 for (i = 0; i < 9; i++) {

 if( i%3 == 0)

 System.out.println(" -----------------------");

 for (j = 0; j < 9; j++) {

 if (j%3 == 0) System.out.print("| ");

 if(boardToPrint[i][j] == 0)

 System.out.print("\* ");

 else

 System.out.print(Integer.toString(boardToPrint[i][j])+ " ");

 }

 System.out.println("|");

 }

 System.out.println(" -----------------------");

 }

}

# Appendix II: Raw data

Here is the raw data gathered by the experiment. This first row, in bold, indicated the number of threads used in the test run. The second column is the average time it took to solve the puzzle whiles the experiment. The following columns are the individual time data harvested by repeatedly performing the test.

Note: due to the random nature of the data collected, repeating the experiment should yield a similar average, but different individual values.

**1 2 3 4 5 6 7 8 9 10**

290 158 242 292 190 105 133 96 113 109

Time (ms)

1124 15 31 265 15 15 16 16 16 16

15 15 16 47 16 15 15 15 15 15

78 16 94 16 15 16 47 31 16 16

78 16 78 16 15 15 16 15 16 15

375 15 31 16 32 16 16 31 31 19

15 16 31 124 1515 172 16 31 281 702

31 15 62 15 15 15 31 16 140 172

16 15 156 31 16 16 16 16 31 16

16 78 32 16 15 16 15 15 125 15

46 16 63 16 16 15 16 16 15 47

47 16 3167 218 31 16 15 31 15 31

78 63 17 218 31 31 16 16 15 31

78 1357 16 11 16 32 31 15 31 203

31 1030 15 17 16 15 15 16 250 468

62 48 16 795 15 15 1 16 93 17

47 63 15 17 15 16 16 16 78 31

328 62 32 795 31 16 15 16 141 16

15 47 15 1950 16 15 16 15 16 16

78 62 78 16 16 62 15 32 78 15

47 47 187 16 15 16 31 15 16 203

63 156 78 15 16 16 16 16 15 203

31 62 156 17 16 16 16 32 31 15

47 62 670 16 1030 16 15 15 18 1451

15 1108 1077 16 15 16 16 15 15 177

156 1616 16 1616 79 15 16 15 32 16

47 110 32 15 16 32 3214 16 218 16

47 484 32 15 31 31 3214 15 16 31

47 15 15 141 16 140 15 16 16 31

46 63 32 16 16 16 203 16 16 31

32 93 16 47 125 15 31 15 16 31

62 93 15 16 31 16 31 31 15 31

47 47 16 16 16 16 140 15 32 31

47 16 15 31 31 15 47 16 15 31

62 31 16 31 15 16 47 15 15 31

47 16 16 390 125 15 47 15 15 21

47 16 15 561 93 16 47 31 16 132

46 15 16 444 32 281 140 328 15 16

63 15 1341 93 16 1982 32 47 16 15

47 93 265 203 16 312 312 15 15 16

1341 15 265 125 15 16 31 16 15 63

1685 16 249 531 31 16 31 16 16 156

15 16 31 16 15 16 31 140 15 3264

32 281 31 15 15 15 218 31 31 15

78 15 1529 15 31 15 16 31 16 15

156 16 16 16 16 266 31 31 16 32

16 16 16 109 15 281 15 31 31 32

406 15 16 15 500 1888 16 125 16 16

15 16 15 16 1451 32 31 125 15 15

16 16 16 16 499 16 31 31 32 31

15 15 16 327 16 31 16 47 15 15

16 16 110 2075 16 16 31 327 1529 31

140 16 936 2075 16 15 16 327 6 16

31 16 78 16 31 16 16 140 15 31

16 31 47 218 17 109 16 218 156 16

31 203 78 15 16 624 15 93 109 15

63 718 1.93E+02 234 31 421 16 47 15 16

47 687 15 15 15 219 15 109 15 16

156 577 16 16 16 15 16 31 7 16

1029 718 15 16 15 47 16 15 31 16

47 687 951 16 16 15 15 16 16 31

47 577 391 187 15 171 16 15 31 15

47 16 391 343 16 16 15 15 16 2262

16 31 125 377 63 16 16 16 31 15

109 16 32 16 15 15 16 16 31 32

47 15 250 1217 421 141 16 16 172 16

46 16 31 2 203 15 16 15 16 16

47 15 1061 64 171 16 62 16 109 15

63 32 15 15 171 16 31 15 203 15

62 15 15 172 780 16 16 15 93 16

47 16 16 15 16 47 16 16 16 16

47 1653 78 312 62 31 15 16 31 16

62 250 78 94 62 31 15 15 31 15

16 484 78 16 47 78 32 15 433 32

31 16 16 94 16 78 343 15 1872 17

15 31 78 93 31 31 359 16 2 15

33 16 297 78 31 16 78 16 16 16

16 16 436 163 109 124 78 31 15 15

15 16 436 78 156 31 172 16 16 16

15 15 187 32 109 47 203 15 16 436

15 16 1029 78 78 16 561 16 16 16

203 78 125 78 16 125 156 15 31 15

1045 15 125 78 15 16 31 16 16 31

546 16 3074 93 16 93 514 16 16 16

47 16 1607 31 31 234 141 15 15 15

811 15 1607 93 16 16 546 16 16 31

671 16 140 31 15 16 16 15 16 312

1170 15 32 109 32 15 16 16 16 94

16 31 188 31 15 16 172 16 1 140

32 16 171 62 3120 15 16 15 15 31

16 31 63 15 31 15 171 10 16 31

218 109 62 24 47 16 171 10 15 31

1217 171 31 33 16 16 16 15 15 16

438 33 31 33 16 16 16 16 16 31

31 811 109 109 15 62 827 452 15 16

1763 62 109 15 16 406 16 234 16 16

281 1045 94 2028 15 31 16 203 15 15

1311 281 312 16 187 78 15 608 32 15

2215 281 140 58 16 47 15 16 15 15

31 1747 16 47 16 78 31 15 16 16

1763 1108 936 32 156 31 16 31 16 15

281 343 94 32 499 125 16 15 15 15

1311 343 31 31 593 982 15 16 16 31

2215 125 109 640 16 982 16 16 31 16

343 343 16 640 63 15 32 16 46 31

47 125 16 1872 16 32 234 15 31 16

47 125 468 640 16 15 561 15 16 31

47 16 125 1872 32 31 1420 31 31 31

62 15 78 125 15 31 16 1731 16 16

47 16 624 73 16 16 718 733 31 15

2995 109 187 15 16 16 110 62 31 62

63 173 844 16 16 125 15 15 16 31

125 125 156 15 16 31 16 31 15 16

31 125 140 16 16 31 31 31 219 32

343 16 16 31 47 31 15 15 15 15

4134 16 16 31 15 16 16 453 218 15

2512 15 94 16 16 110 281 172 47 16

460 16 250 15 15 78 1529 1248 203 16

460 15 250 16 16 47 15 31 158 16

460 47 16 160 31 16 15 31 31 15

460 281 172 1616 15 47 15 17 31 16

46 265 47 15 16 16 15 171 249 16

16 1077 48 15 15 16 437 359 32 15

156 142 31 16 16 16 15 359 31 16

15 1139 78 16 16 15 296 670 15 1498

16 94 62 15 16 16 16 31 32 193

15 15 32 15 15 15 219 15 31 31

32 31 109 16 32 280 15 15 15 31

15 78 16 327 16 406 16 15 32 171

16 140 46 3089 31 15 234 16 2636 468

15 16 110 1332 15 31 16 15 2636 16

16 32 297 16 2325 16 47 16 889 16

16 141 109 31 1248 15 156 32 31 16

686 16 109 343 16 16 16 32 31 16

1170 16 312 16 16 109 47 16 156 15

31 78 109 15 15 187 16 31 16 296

125 47 109 32 16 32 31 687 78 296

46 16 312 15 31 31 16 32 390 31

16 78 172 15 64 31 15 172 16 164

16 47 172 16 94 141 47 78 16 842

15 15 172 62 140 187 173 31 15 15

16 63 592 62 15 31 31 16 16 33

15 15 187 16 15 47 15 16 11 15

16 47 1264 15 156 31 94 31 1515 15

16 62 94 16 265 47 62 15 16 16

15 32 2293 16 844 31 94 15 16 16

16 31 16 16 125 828 62 1763 15 16

15 31 125 15 780 16 94 15 16 47

17 47 78 15 31 31 62 31 15 15

31 46 140 47 140 31 546 172 31 16

16 47 951 16 156 31 32 171 15 47

15 31 171 16 2980 31 16 16 62 421

2278 47 140 15 187 203 15 31 15 905

1045 47 219 32 187 78 16 64 16 218

655 297 93 1467 2200 16 15 15 15 16

172 282 47 486 31 31 16 16 16 109

187 280 79 904 125 1616 78 156 15 16

375 16 78 170 3089 78 16 406 15 31

125 16 94 203 47 78 156 15 15 15

124 31 78 1341 16 31 32 16 16 16

78 16 140 31 16 111 16 16 15 16

125 171 31 345 16 16 78 15 16 47

125 31 93 16 15 16 47 16 15 31

16 47 93 1186 32 16 47 16 15 16

78 64 156 15 31 16 125 1763 16 15

1185 46 2168 15 16 15 109 15 16 15

124 47 48 16 156 16 31 31 47 16

390 62 62 1515 390 16 31 172 31 16

15 78 62 16 889 15 187 171 16 63

327 47 32 124 889 16 16 16 15 31

171 62 31 359 16 31 31 31 16 15

203 47 15 254 31 31 15 64 530 16

187 31 16 125 140 31 327 15 196 31

47 16 16 3123 156 31 374 16 1045 31

47 171 15 15 2980 203 16 156 16 218

47 48 16 15 187 78 78 406 16 62

111 47 16 31 187 16 16 15 16 15

93 16 16 343 2200 31 31 16 15 178

111 15 32 125 31 1616 32 16 16 79

93 16 32 16 125 78 16 15 15 31

16 15 32 15 3089 78 15 16 31 203

15 15 15 231 47 31 16 16 15 15