SOLUTION - Problem Set 7

6.105

Refer to Fig. 6.43.

$I = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}V_{ov}^2 \Rightarrow 100\mu A = \frac{1}{2}\times 60\frac{W}{L}0.2^2$

$\Rightarrow \frac{W}{L} = 83.3$

$V_{SG1} = V_{DD} - V_{BIAS1} = 3.3 - V_{BIAS1}$

$V_{ov} = V_{SG1} - |V_{tp}| = 0.2 \Rightarrow 0.2 = 3.3 - V_{BIAS1} - 0.8$

$\Rightarrow \underline{V_{BIAS1} = 2.3\,V}$

For Maximum swing: $V_{SD1} = V_{ov} \Rightarrow V_{D1} = 3.3 - 0.2 = 3.1^V$

$\Rightarrow V_{D1} = 3.1^V$

then: $V_{SG2} - |V_{tp}| = V_{ov} \Rightarrow 3.1 - V_{BIAS2} - 0.8 = 0.2$

$\underline{V_{BIAS2} = 2.1V}$

The highest allowable voltage at the output is $V_{DD} - V_{ov} - V_{ov} = 3.3 - 0.2 - 0.2 = \underline{2.9^V}$

$R_o \simeq g_{m2}\, r_{o2}\, r_{o1}$ (Eq. 6.141)

$g_m = \frac{2I_D}{V_{ov}} = \frac{200\mu}{0.2} = 1\,mA/V$ $\quad r_o = \frac{V_A}{I_D} = \frac{5}{100\mu} = 50k\Omega$

$R_o = 4\times 50\times 50 = 2.5M\Omega$

$\underline{R_o = 2.5M\Omega}$

Solution - Problem Set 7 (contd.)

Problem 1

Figure 1

A)

$$I_{E1,2} = \frac{I}{2}$$

$$I_{E3,4} = I_{E1,2} = \frac{I}{2}$$

B)

- $$R_i = (\beta+1)(2 r_{e1,2})$$

- Assume $V_{A,npn} = V_{A,pnp} \Rightarrow r_{o,npn} = r_{o,pnp} \triangleq r_o$

$$R_o = R_{o4} \parallel R_{o7}$$

where

$$R_{o7} = \frac{\beta}{2} r_o \quad \text{Wilson current mirror}$$

$$R_{o4} \simeq r_o (1+\beta) \simeq \beta r_o \quad \text{assuming } r_o \gg r_\pi$$

$$\Rightarrow R_o = \frac{\beta}{3} r_o$$

Figure 2

A)

$$I_{E1,2} = \frac{I}{2}$$

$$I_{E3,4} = \frac{3}{4} I - I_{E1,2} = \frac{1}{4} I$$

B)

- $R_i = (B+1)(2 r_{e1,2})$
- Assume $V_{A,npn} = V_{A,pnp} \Rightarrow r_{o,npn} = r_{o,pnp} \triangleq r_o$

$$R_o = R_{o4} \| R_{o5}$$

where

$$R_{o5} = \frac{\beta}{2} r_o \qquad \text{Wilson current mirror}$$

$$R_{o4} = r_{o4}\left[1 + g_{m4}\left((r_{o2} \| R_L) \| r_{\pi 4}\right)\right]$$ where R_L is the output resistance of source $\left(\frac{3}{4} I\right)$

$$\simeq r_o(1+\beta) \simeq \beta r_o \quad \text{assuming } R_L \gg r_o \text{ and } r_o \gg r_\pi$$

$$\Rightarrow R_o = \frac{\beta}{3} r_o$$

Figures 1 and 2

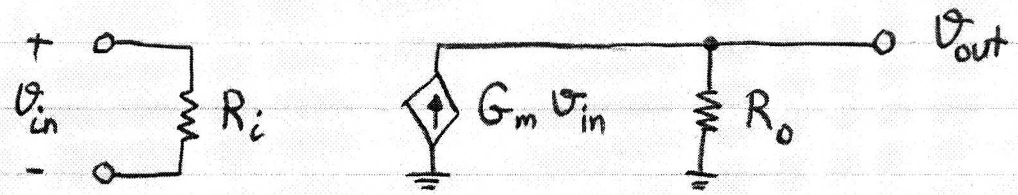
B) (contd.)

- $$i_e = \frac{v_{in}}{2\, r_{e1,2}} \Rightarrow i_c = \alpha\, i_e = \frac{1}{2}\, g_{m1,2}\, v_{in}$$

$$\Rightarrow i_{out} = 2\, i_c = g_{m1,2}\, v_{in}$$

$$\Rightarrow G_m \triangleq \left.\frac{i_{out}}{v_{in}}\right|_{R_L=0} = g_{m1,2}$$

Equivalent Transconductance Circuit Model:



- $$A_v \triangleq \left.\frac{v_{out}}{v_{in}}\right|_{R_L=\infty} = G_m R_o = (g_{m1,2})\left(\frac{1}{3}\beta r_o\right)$$

Figure 1

c)

- $\underline{V_{BIAS,min}} = \underline{V_{BE\,on} + V_{CE\,sat} + V_{CS} - V_{EE}}$

- $V_{BC1,2} = V_{IN} - (V_{BIAS} - V_{BE\,on\,3,4}) \leqslant V_{BC\,on1,2}$

$\Rightarrow \underline{V_{IN\,max}} = V_{BIAS} - V_{BE\,on} + V_{BC\,on} = \underline{V_{BIAS} - V_{CE\,sat}}$

- $V_{BE1,2} = V_{IN} - (-V_{EE} + V_{CS}) \geqslant V_{BE\,on}$

$\Rightarrow \underline{V_{IN\,min}} = \underline{-V_{EE} + V_{CS} + V_{BE\,on}}$

- $V_{CB7} = V_0 - (V_{CC} - |V_{EB\,on6}| - |V_{EB\,on7}|) \leqslant |V_{CB\,on7}|$

$\Rightarrow \underline{V_{OUT\,max}} = V_{CC} - |V_{EC\,sat7}| - |V_{EB\,on6}| = \underline{V_{CC} - (V_{BE\,on} + V_{CE\,sat})}$

- $V_{BC4} = V_{BIAS} - V_0 \leqslant V_{BC\,on4}$

$\Rightarrow \underline{V_{OUT\,min}} = \underline{V_{BIAS} - V_{BC\,on}}$

Figure 2

c)

- $V_{BIAS,max} = V_{CC} - V_{CS} - V_{BEon}$

- $V_{BC1,2} = V_{IN} - \left(V_{BIAS} + |V_{EBon3,4}|\right) \leqslant V_{BCon1,2}$

$\Rightarrow V_{INmax} = V_{BIAS} + V_{BEon} + V_{BCon} = V_{BIAS} + 2V_{BEon} - V_{CEsat}$

- $V_{BE1,2} = V_{IN} - (-V_{EE} + V_{CS}) \geqslant V_{BEon1,2}$

$\Rightarrow V_{INmin} = -V_{EE} + V_{CS} + V_{BEon}$

- $V_{CB4} = V_{OUT} - V_{BIAS} \leqslant |V_{CBon4}|$

$\Rightarrow V_{OUTmax} = V_{BIAS} + V_{BCon}$

- $V_{BC5} = \left(V_{BEon5} + V_{BEon7} - V_{EE}\right) - V_{OUT} \leqslant V_{BCon5}$

$\Rightarrow V_{OUTmin} = -V_{EE} + V_{BEon} + V_{CEsat}$

C) (contd.)

$\therefore$ If $V_{CS} = \left| V_{CEsat} + V_{BEon} \right|$, then for proper operation:

Figure 1 : Cascode Amplifier

$$\boxed{\begin{array}{c} -V_{EE} + 2V_{BEon} + 2V_{CEsat} \leqslant V_{BIAS} \\ -V_{EE} + 2V_{BEon} + V_{CEsat} \leqslant V_{IN} \leqslant V_{BIAS} - V_{CEsat} \\ V_{BIAS} - V_{BEon} + V_{CEsat} \leqslant V_{OUT} \leqslant V_{CC} - (V_{BEon} + V_{CEsat}) \end{array}}$$

Figure 2 : Folded-Cascode Amplifier

$$\boxed{\begin{array}{c} V_{BIAS} \leqslant V_{CC} - 2V_{BEon} - V_{CEsat} \\ -V_{EE} + 2V_{BEon} + V_{CEsat} \leqslant V_{IN} \leqslant V_{BIAS} + 2V_{BEon} - V_{CEsat} \\ -V_{EE} + V_{BEon} + V_{CEsat} \leqslant V_{out} \leqslant V_{BIAS} + V_{BEon} - V_{CEsat} \end{array}}$$

Problem 2

a) $V_{OUT\,max} = V_{CC} - (V_{BE\,on} + V_{CE\,sat})$

$= 5 - (0.7 + 0.3) = \underline{\underline{4V}}$

b) $V_{OUT\,bias} = V_{OUT\,max} - 1.5V = 4 - 1.5 = \underline{\underline{2.5V}}$

c) $V_{OUT\,min} = V_{OUT\,bias} - 1.5V = \underline{\underline{1V}}$

$V_{BIAS} \leqslant V_{out\,min} + V_{BC\,on} = 1 + 0.4 = \underline{\underline{1.4V}}$

d) $V_{IN\,max} = V_{BIAS} - V_{CE\,sat} = V_{BIAS} - (V_{BE\,on} - V_{BC\,on})$

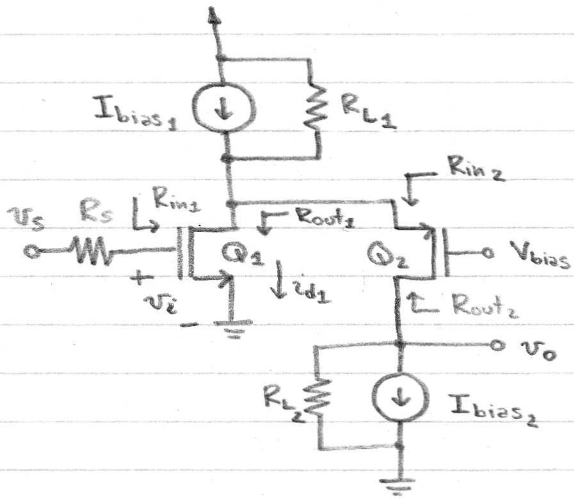
$= 1.4 - (0.7 - 0.4)$

$= \underline{\underline{1.1V}}$

PROBLEM 3

MOS Folded Cascode Amplifier

i) Midband voltage gain:



$$i_{d1} = g_{m1} v_i = g_{m1} v_s \quad (R_{in1} \to \infty)$$

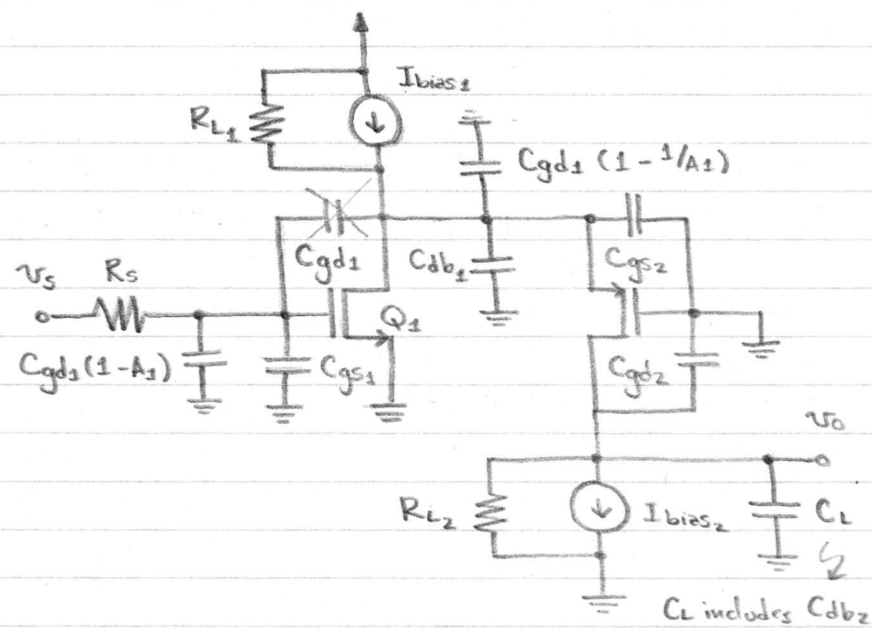
$$v_o = -i_{d1} (R_{out2} \| R_{L2})$$

$$\to R_{out2} \triangleq g_{m2} r_{o2} (r_{o1} \| R_{L1})$$

$$A_M = \frac{v_o}{v_s} = -g_{m1} (R_{out2} \| R_{L2})$$

ii) 3dB frequency:

Use Miller's theorem to split C_{gd1} into two caps. to ground...



$$A_1 = -g_{m1} (R_{out1} \| R_{in2} \| R_{L1})$$

$$R_{out1} = r_{o1}$$

$$R_{in2} \triangleq \frac{1}{g_{m2}} \left(1 + \frac{R_{L2}}{r_{o2}}\right)$$

@ the gate of Q_1

$C_{g1} = C_{gs1} + C_{gd1}(1 - A_1)$

$R_{g1} = R_s$

$\tau_{g1} = R_{g1} C_{g1}$

@ the drain of Q_1

$C_{d1} = C_{db1} + C_{gd1}(1 - 1/A_1) + C_{gs2}$

$R_{d1} = R_{in2} \| r_{o1} \| R_{L1}$

$\tau_{d1} = R_{d1} C_{d1}$

@ the drain of Q_2

$C_{d2} = C_L + C_{gd2}$

$R_{d2} = R_{out2} \| R_{L2}$

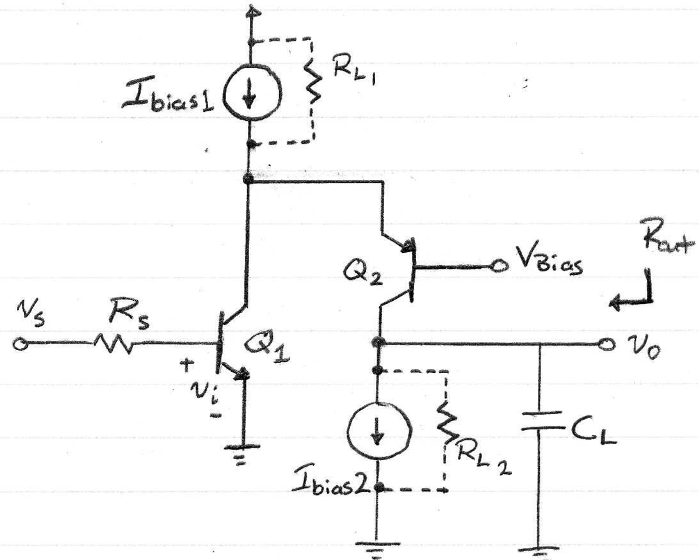
$\tau_{d2} = R_{d2} C_{d2}$

Assuming a dominant pole exists, the 3dB frequency can be derived as:

$$\omega_H = \frac{1}{\tau_{g1} + \tau_{d1} + \tau_{d2}}$$

BJT Folded_Cascode Amplifier

i) Midband Voltage Gain



- $v_i = \dfrac{r_{\pi_1}}{R_s + r_{\pi_1}} v_s$

where $r_{\pi_1} = (\beta+1) r_{e_1}$

- Short-Circuit Transconductance

$$G_m \triangleq \left.\frac{i_o}{v_s}\right|_{v_o=0} = \frac{-\alpha g_m v_i}{\frac{R_s + r_{\pi_1}}{r_{\pi_1}} v_i} = \frac{-\alpha g_{m_1}}{1 + R_s/r_{\pi_1}}$$

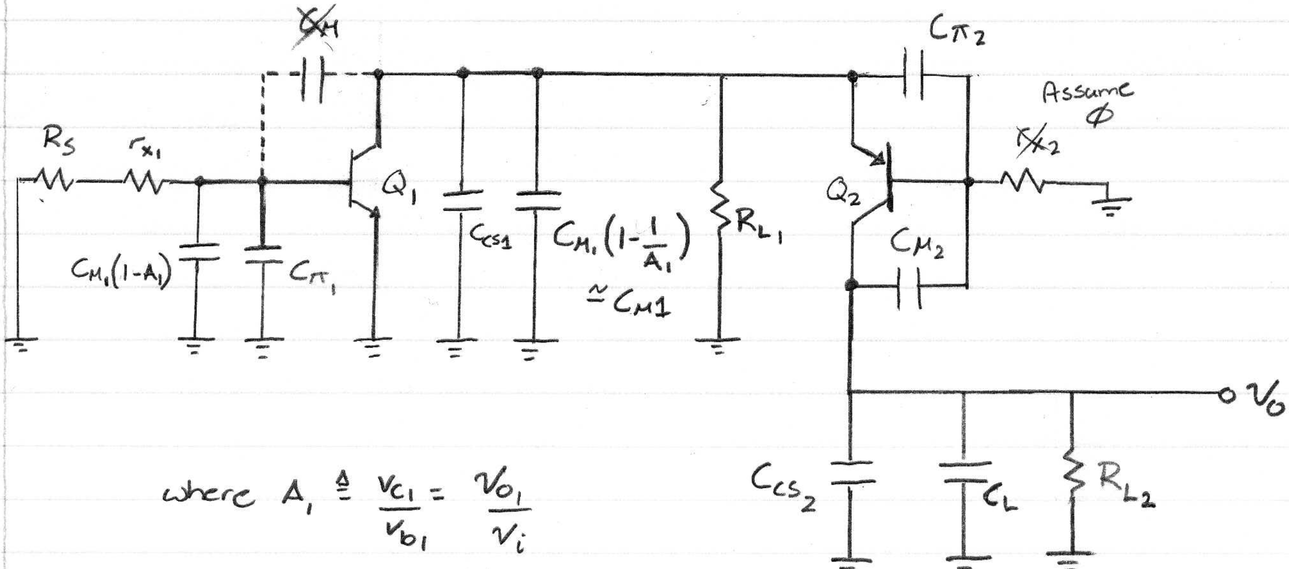
- $R_{out} = R_{L_2} \parallel r_{o_2}\left[1 + g_{m_2}\left(R_{L_1} \parallel r_{o_1} \parallel r_{\pi_2}\right)\right]$

- Midband Voltage Gain

$$A_M \triangleq \frac{v_o}{v_s} = G_m R_{out}$$

$$= \frac{-\alpha g_{m1}}{1 + R_s/r_{\pi_1}} \; R_{L_2} \parallel r_{o_2}\left[1 + g_{m_2}\left(R_{L_1} \parallel r_{o_1} \parallel r_{\pi_2}\right)\right]$$

ii) 3 dB Frequency



where $A_1 \triangleq \frac{v_{c1}}{v_{b1}} = \frac{V_{o1}}{V_i}$

$$= -g_{m1}\left(r_{o1} \| R_{L1} \| r_{e2}\left[\frac{r_{o2}+R_{L2}}{r_{o2}+R_{L2}/(\beta_2+1)}\right]\right)$$

1) Apply Miller's theorem

$\Rightarrow$ All capacitors now connected between a node and GND (shown on diagram)

2) Find Open-Circuite Time Constants associated with each node:

- C_2 (output) node

$C_{c2} = C_{cs2} + C_L + C_{\mu 2}$

Resistance seen: $R_{c2} = R_{L2} \,\|\, r_{o2}[1+g_{m2}(R_L \| r_{o1} \| r_{\pi 2})]$

$\Rightarrow \omega_{c2} = \frac{1}{\tau_{c2}} = 1/(C_{c2}R_{c2})$

- C_1 node

$$C_{C1} = C_{\pi_2} + C_{cs1} + C_{\mu 1}\left(1 - \frac{1}{A_1}\right)$$

Resistance seen:

$$R_{C1} = r_{o1} \,//\, R_{L1} \,//\, r_{e2}\left[\frac{r_{o2} + R_{L2}}{r_{o2} + R_{L2}/(B_2+1)}\right]$$

$$\Rightarrow \omega_{C1} = \frac{1}{\tau_{C1}} = 1/(C_{C1} R_{C1})$$

- B1 node

$$C_{B1} = C_{\mu 1}(1 - A_1) + C_{\pi 1}$$

Resistance seen: $R_{B1} = r_{\pi_1} \,//\, (r_x + R_S)$

$$\Rightarrow \omega_{B1} = 1/\tau_{B1} = 1/(C_{B1} R_{B1})$$

3) Find ω_H

Assuming a Dominant Pole exists

$$\omega_H = \frac{1}{\tau_{C2} + \tau_{C1} + \tau_{B1}}$$