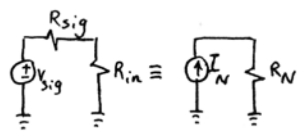
PROBLEM SET 6 - SOLUTION

Problem1

6.54

a) $R_{in} = \frac{R}{1-A} = \frac{R}{1-2} = -R$ (Miller's theorem)

b) $I_N = \frac{V_{sig}}{R_{sig}}$



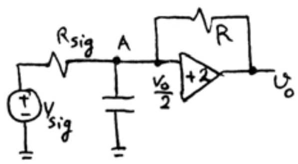
$R_N = R_{sig} // R_{in}$

IF $R_{sig} = R$ then :

$R_N = R // (-R) = \infty \Rightarrow I_L = I_N = \frac{V_{sig}}{R_{sig}} = \frac{V_{sig}}{R}$

Cont.

KCL at A :



$$\frac{\frac{V_o}{2} - V_{sig}}{R_{sig}} + \frac{v_o}{2} \times CS + \frac{-v_o}{2R} = 0$$

IF $R_{sig} = R \Rightarrow \frac{+v_{sig}}{R} = \frac{v_o}{2} CS \Rightarrow \frac{v_o}{v_{sig}} = \frac{2}{RCS}$

Problem 2

6.99

a) $I = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 \Rightarrow$ For same I: $\frac{V_{ov_b}^2}{V_{ov_a}^2} = \frac{(\frac{W}{L})_a}{(\frac{W}{L})_b}$

For same I, if $\frac{W}{L}$ is divided by 4, then V_{ov}^2 is multiplied by 4, or equivalently

Cont.

V_{ov} is doubled.

$g_m = \mu_n C_{ox} \frac{W}{L} V_{ov}$. Thus g_m for circuit (b) is half of the one for circuit (a).

$A_o = g_m r_o = \frac{2I_D}{V_{ov}} \times \frac{V_A}{I_D} = \frac{2V_A' L}{V_{ov}}$. Thus if L is multiplied by 4 and V_{ov} is halved, then A_o is doubled for circuit (b).

In summary, for circuit (b), V_{ov} is doubled, g_m is halved, A_o is doubled.

b) Each transistor in circuit (c) has the same overdrive voltage as the one in circuit (a). Referring to Eq. 6.129 and 6.130:

$A_{v_o} = -A_o^2 = -(g_m r_o)^2$

$G_m \simeq g_{m1} = g_m$ (Same as circuit (a))

Note that for the transistors in circuit (c) the g_m and r_o are the same as the ones in circuit (a). Thus the intrinsic gain for circuit (c), $A_{v_o} = -A_o^2$ where A_o is the intrinsic gain for circuit (a).

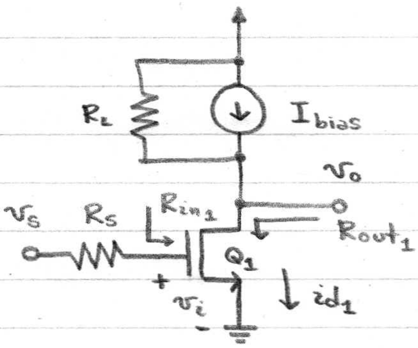
In general, circuit (c) has a higher output resistance and for the same V_{ov} of transistors it has lower output swing. The output swing is limited to $2V_{ov}$ on the low side for circuits (b) and (c) while it is only limited to V_{ov} for circuit (a).

PROBLEM 3

MOS Amplifiers

(a) MOS Common Source Amplifier :

i) Midband voltage gain:



$$v_i = \frac{R_{in_1}}{R_s + R_{in_1}} v_s \doteq v_s \quad (\text{since } R_{in_1} \to \infty)$$

$$i_{d1} = g_{m_1} v_i = g_{m_1} v_s$$

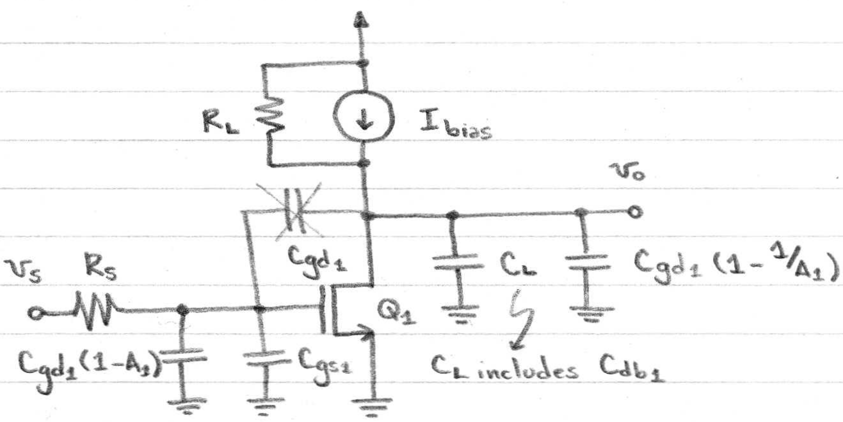
$$v_o = -i_{d_1}(R_{out_1} \| R_L) = -g_{m_1}(r_{o_1} \| R_L) v_s$$

$$A_M = \frac{v_o}{v_s} = -g_{m_1}(r_{o_1} \| R_L)$$

ii) 3dB frequency:

Use Miller's theorem to split C_{gd_1} into two caps. to ground ...

$A_1 = A_M$ (derived above)



@ the gate of Q_1

$C_{g_1} = C_{gs_1} + C_{gd_1}(1 - A_1)$

$R_{g_1} = R_s$

$\tau_{g_1} = R_{g_1} C_{g_1}$

@ the drain of Q_1

$C_{d_1} = C_L + C_{gd_1}(1 - 1/A_1)$

$R_{d_1} = R_{out_1} \| R_L = r_{o_1} \| R_L$

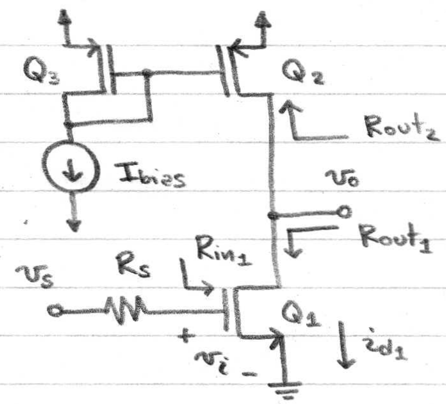
$\tau_{d_1} = R_{d_1} C_{d_1}$

Using the open circuit time constants τ_{g_1} & τ_{d_1}, and assuming a dominant pole exists, the 3dB frequency can be derived as:

$$\omega_H = \frac{1}{\tau_{g_1} + \tau_{d_1}}$$

(b) MOS Common-Source Amplifier with Active Load:

i) Midband voltage gain:

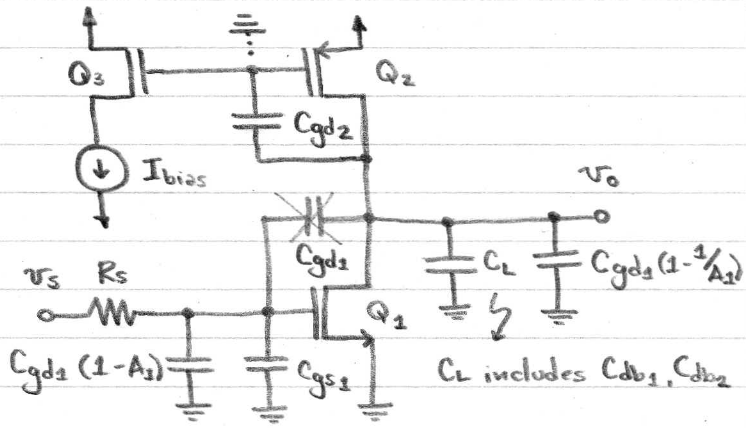


$$i_{d_1} = g_{m_1} v_i = g_{m_1} v_s \qquad (R_{in_1} \to \infty)$$

$$v_o = -i_{d_1} (R_{out_1} \| R_{out_2}) = -g_{m_1} (r_{o_1} \| r_{o_2}) v_s$$

$$A_M = \frac{v_o}{v_s} = -g_{m_1} (r_{o_1} \| r_{o_2})$$

ii) 3dB frequency:



Use Miller's theorem to split C_{gd_1} into two caps. to ground...

$A_1 = A_M$ (derived above)

Since V_{G2} is signal ground, C_{gd_2} is connected between v_o and ground.

All remaining capacitances ($C_{gs_3}, C_{gd_3}, C_{db_3}, C_{gs_2}$) are outside the signal path and do not affect the response.

@ the gate of Q_1

$$C_{g_1} = C_{gs_1} + C_{gd_1}(1-A_1)$$
$$R_{g_1} = R_s$$
$$\tau_{g_1} = R_{g_1} C_{g_1}$$

@ the drain of Q_1

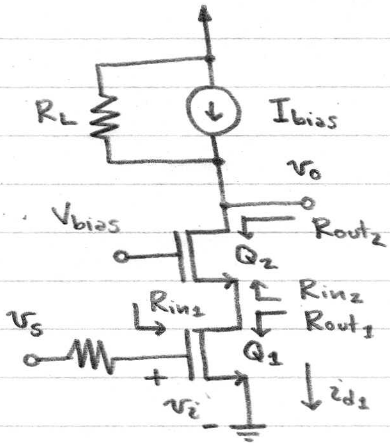
$$C_{d_1} = C_L + C_{gd_2} + C_{gd_1}(1 - 1/A_1)$$
$$R_{d_1} = R_{out_1} \| R_{out_2} = r_{o_1} \| r_{o_2}$$
$$\tau_{d_1} = R_{d_1} C_{d_1}$$

Assuming a dominant pole exists, the 3dB frequency can be derived as:

$$\omega_H = \frac{1}{\tau_{g_1} + \tau_{d_1}}$$

(c) MOS Cascode Amplifier :

i) Midband voltage gain:



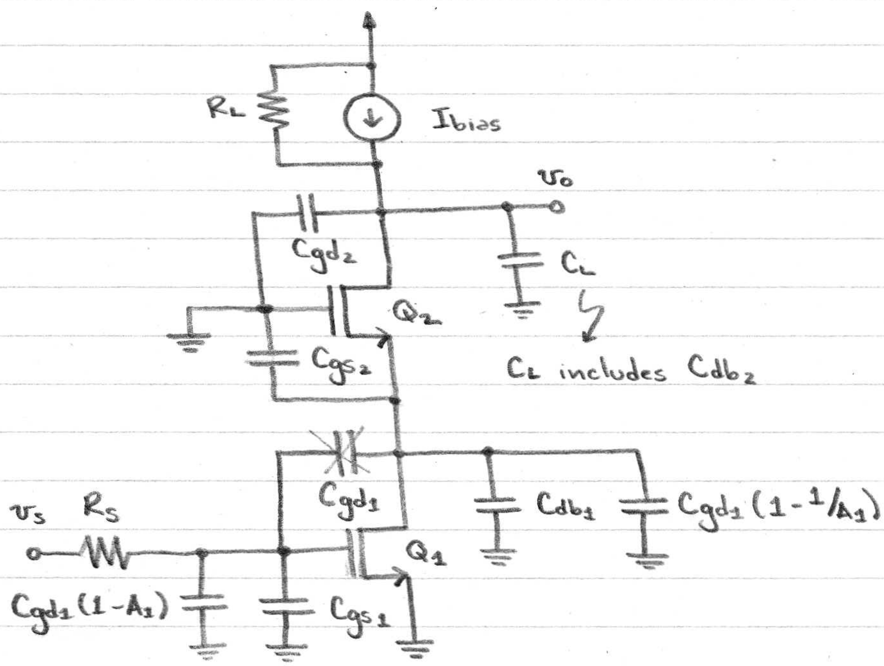
$i_{d1} = g_{m1} v_i = g_{m1} v_s \quad (R_{in1} \to \infty)$

$v_o = -i_{d1} (R_{out2} \| R_L) \qquad R_{out2} \simeq g_{m2} r_{o2} r_{o1}$

$$A_M = \frac{v_o}{v_s} = -g_{m1} (R_{out2} \| R_L)$$

ii) 3dB frequency:

Use Miller's theorem to split C_{gd1} into two caps. to ground...



$A_1 = -g_{m1} (R_{out1} \| R_{in2})$

$R_{out1} = r_{o1}$

$$R_{in2} \simeq \frac{1}{g_{m2}} \left(1 + \frac{R_L}{r_{o2}}\right)$$

@ the gate of Q_1

$C_{g1} = C_{gs1} + C_{gd1}(1 - A_1)$

$R_{g1} = R_S$

$\tau_{g1} = R_{g1} C_{g1}$

@ the drain of Q_1

$C_{d1} = C_{db1} + C_{gd1}(1 - 1/A_1) + C_{gs2}$

$R_{d1} = R_{in2} \| r_{o1}$

$\tau_{d1} = R_{d1} C_{d1}$

@ the drain of Q_2

$C_{d2} = C_L + C_{gd2}$

$R_{d2} = R_{out2} \| R_L$

$\tau_{d2} = R_{d2} C_{d2}$

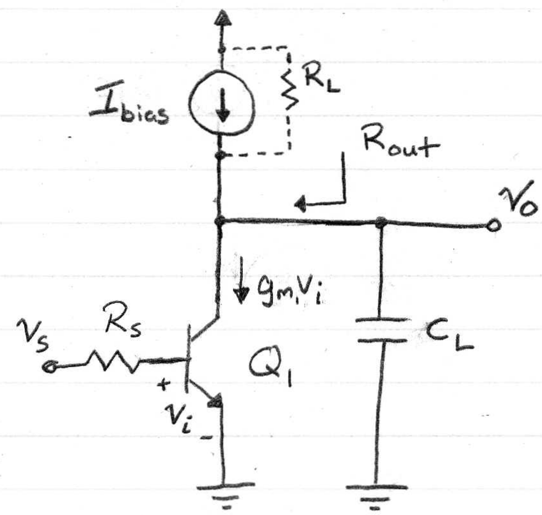
Assuming a dominant pole exists, the 3dB frequency can be derived as:

$$\omega_H = \frac{1}{\tau_{g1} + \tau_{d1} + \tau_{d2}}$$

BJT Amplifiers

(a) BJT Common-Emitter Amplifier

i) Midband Voltage Gain



- $$v_i = \frac{r_{\pi_1}}{R_s + r_{\pi_1}} v_s$$

  where $r_{\pi_1} = (\beta+1) r_{e_1}$

- Short-Circuit Transconductance:

$$G_m \triangleq \left.\frac{i_o}{v_s}\right|_{v_o=0} = \frac{-g_{m_1} v_i}{\frac{R_s + r_{\pi_1}}{r_{\pi_1}} v_i} = \frac{-g_{m_1}}{1 + R_s/r_{\pi_1}}$$

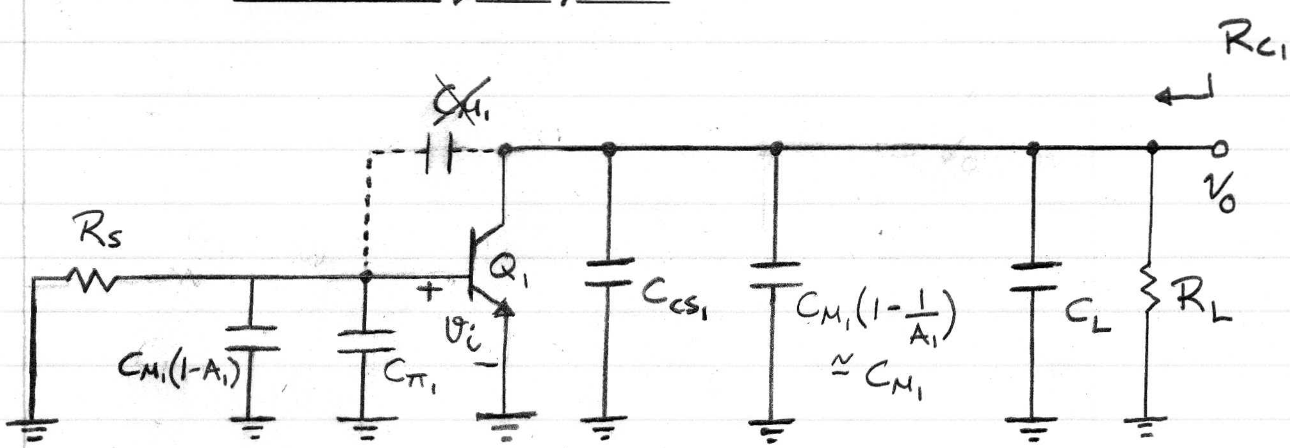
- $R_{out} = R_L \,//\, r_{o_1}$

- Midband Voltage Gain:

$$A_M \triangleq \frac{v_o}{v_s} = G_m R_{out}$$

$$= \frac{-g_{m_1}}{1 + R_s/r_{\pi_1}} R_L \,//\, r_{o_1}$$

ii) 3 dB Frequency



where $A_1 \triangleq \dfrac{v_{c1}}{v_{b1}} = \dfrac{V_o}{V_i} = -g_{m_1}(r_{o1} \,//\, R_L)$

1) Apply Miller's theorem

⇒ All capacitors are now connected between a node and GND (shown on diagram)

2) Find Open-Circuit Time Constants associated with each node:

- C1 (output) node

$$C_{c_1} = C_{cs_1} + C_{M_1}\left(1 - \frac{1}{A_1}\right) + C_L$$

Resistance seen: $R_{c_1} = r_{o_1} \,//\, R_L$

$$\omega_{c_1} = 1/\tau_{c_1} = 1/(C_{c_1} R_{c_1})$$

- **B1 node**

$$C_{B1} = C_{\mu_1}(1 - A_1) + C_{\pi_1}$$

Resistance seen: $R_{B_1} = r_{\pi_1} \parallel R_S$

$$\Rightarrow \omega_{B1} = 1/\tau_{B1} = 1/(C_{B1} R_{B1})$$

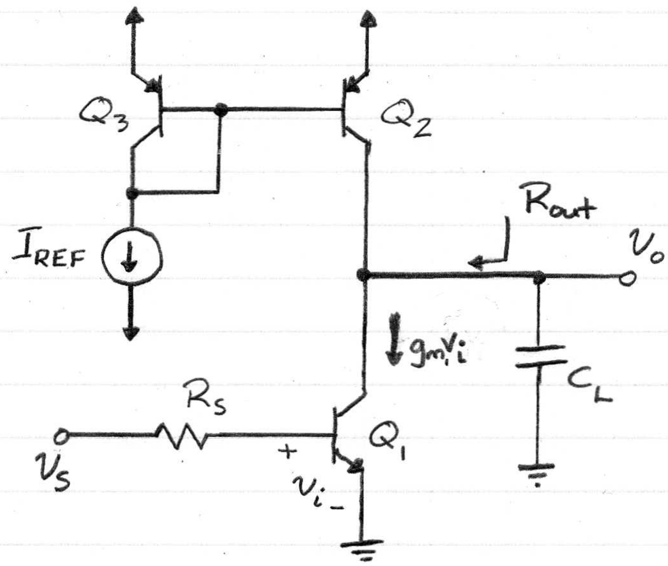
3) Find ω_H

Assuming a Dominant Pole exists:

$$\omega_H = \frac{1}{\tau_{c1} + \tau_{B1}}$$

(b) BJT Common-Emitter Amplifier with Active Load

i) Midband Voltage Gain



- $$v_i = \frac{r_{\pi_1}}{R_s + r_{\pi_1}} v_s$$

  where
  $$r_{\pi_1} = (\beta+1) r_{e_1}$$

- Short-Circuit Transconductance

$$G_m \triangleq \left. \frac{i_o}{v_s} \right|_{v_o=0} = \frac{-g_{m_1} v_i}{\frac{R_s + r_{\pi_1}}{r_{\pi_1}} v_i} = \frac{-g_{m_1}}{1 + R_s/r_{\pi_1}}$$

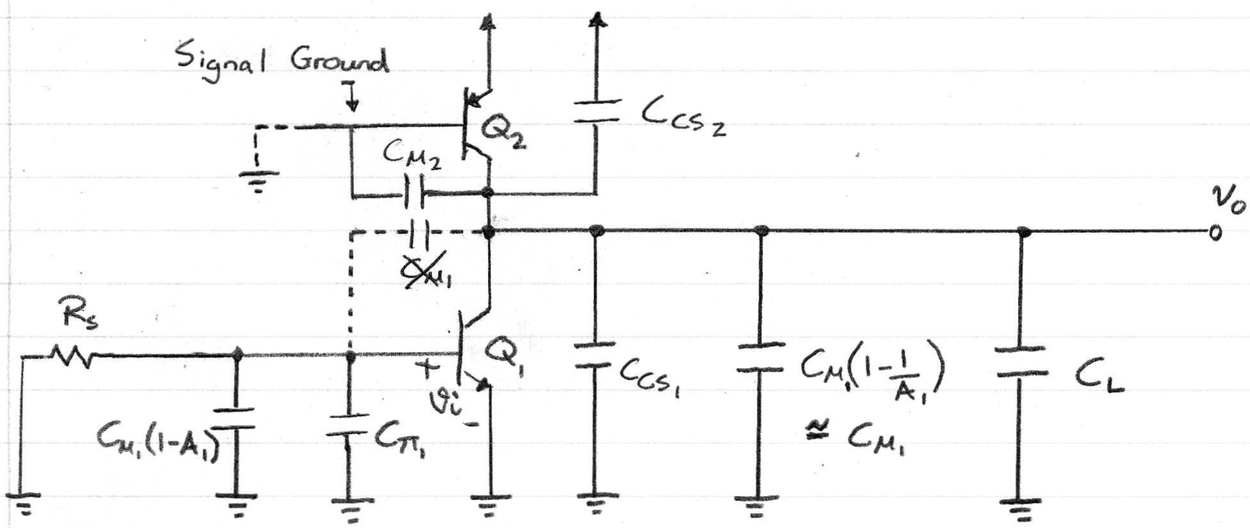
- $R_{out} = r_{o_2} \parallel r_{o_1}$

- Midband Voltage Gain

$$A_M \triangleq \frac{v_o}{v_s} = G_m R_{out}$$

$$= \frac{-g_{m_1}}{1 + R_s/r_{\pi_1}} \; r_{o_2} \parallel r_{o_1}$$

ii) 3dB Frequency



where $A_1 \triangleq \dfrac{v_{c1}}{v_{B1}} = \dfrac{v_o}{v_i} = -g_{m_1}(r_{o1} \parallel r_{o2})$

1) Apply Miller's theorem

$\Rightarrow$ All capacitors are now connected between a node and GND (shown on diagram)

2) Find Open-Circuit Time Constants associated with each node:

- C1 (output) node

$$C_{C1} = C_{CS1} + C_{\mu_1}\left(1 - \frac{1}{A_1}\right) + C_L + C_{CS2} + C_{\mu 2}$$

Resistance seen: $R_{C_1} = r_{o_1} \,//\, r_{o_2}$

$$\Rightarrow \omega_{C1} = 1/\tau_{C_1} = 1/(C_{C1} R_{C1})$$

- B1 node

$$C_{B1} = C_{\mu_1}(1 - A_1) + C_{\pi_1}$$

Resistance seen: $R_{B_1} = r_{\pi_1} \,//\, R_S$

$$\Rightarrow \omega_{C2} = 1/\tau_{C2} = 1/(C_{B1} R_{B1})$$

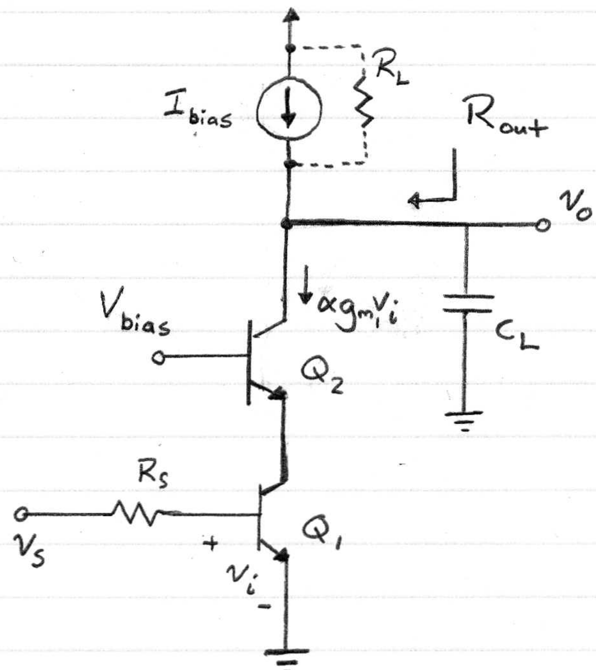
3) Find ω_H

Assuming a Dominant Pole exists:

$$\omega_H = \frac{1}{\tau_{C_1} + \tau_{B1}}$$

(c) BJT Cascode Amplifier

i) Midband Voltage Gain



- $v_i = \dfrac{r_{\pi_1}}{R_s + r_{\pi_1}} v_s$

  where $r_{\pi_1} = (\beta+1) r_{e_1}$

- Short-Circuit Transconductance

$$G_m \triangleq \left.\frac{i_o}{v_s}\right|_{v_o=0} = \frac{-\alpha g_{m_1} v_i}{\frac{R_s + r_{\pi_1}}{r_{\pi_1}} v_i} = \frac{-\alpha g_{m1}}{1 + R_s/r_{\pi_1}}$$

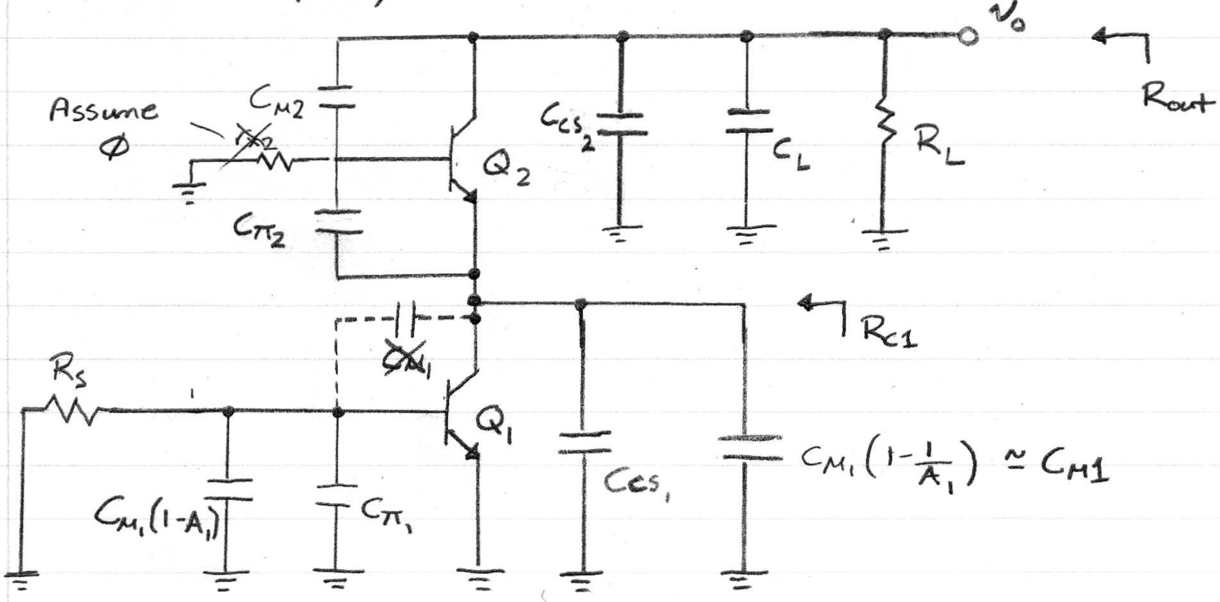
- $R_{out} = R_L \,/\!/\, \beta_2 r_{o2}$

- Midband Voltage Gain

$$A_M \triangleq \frac{v_o}{v_s} = G_m R_{out}$$

$$= \frac{-\alpha g_{m1}}{1 + R_s/r_{\pi_1}} \; R_L \,/\!/\, \beta_2 r_{o2}$$

ii) 3 dB Frequency



where $A_1 \triangleq \dfrac{v_{C1}}{v_{B1}} = \dfrac{v_{o1}}{v_i} = -g_{m_1}\left(r_{o_1} \,//\, r_{e_2}\left[\dfrac{r_{o_2} + R_L}{r_{o_2} + R_L/(\beta_2+1)} \right]\right)$

1) Apply Miller's theorem
   ⇒ All capacitors now connected between a node and GND (shown on diagram)

2) Find Open-Circuit Time Constants associated with each node:

- C_2 (output) node

$$C_{C2} = C_{CS2} + C_{M2} + C_L$$

Resistance seen: $R_{C2} = R_L \,//\, \beta_2 r_{o_2}$

$$\Rightarrow \omega_{C2} = 1/\tau_{C2} = 1/(C_{C2} R_{C2})$$

- C1 node

$$C_{C1} = C_{\pi 2} + C_{CS1} + C_{\mu 1}\left(1 - \frac{1}{A_1}\right)$$

Resistance seen:

$$R_{C1} = r_{o1} \,\|\, r_{e2}\left[\frac{r_{o2} + R_L}{r_{o2} + R_L/(\beta_2+1)}\right]$$

$$\Rightarrow \omega_{C1} = \frac{1}{\tau_{C1}} = 1/(C_{C1} R_{C1})$$

- B1 node

$$C_{B1} = C_{\mu 1}(1 - A_1) + C_{\pi 1}$$

Resistance seen: $R_{B1} = r_{\pi 1} \,//\, R_S$

$$\Rightarrow \omega_{B1} = 1/\tau_{B1} = 1/(C_{B1} R_{B1})$$

3) Find ω_H

Assuming a Dominant Pole Exists

$$\omega_H = \frac{1}{\tau_{C2} + \tau_{C1} + \tau_{B1}}$$

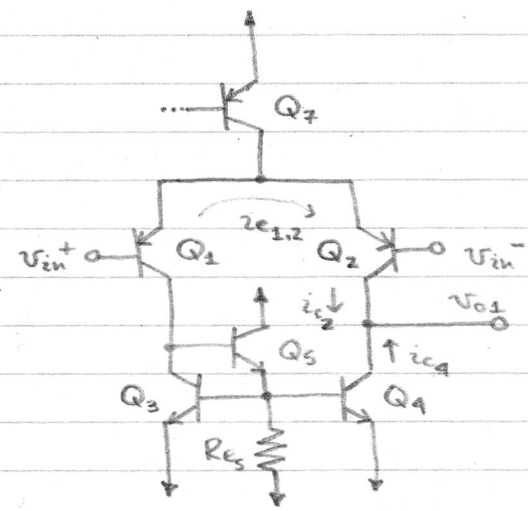
PROBLEM 4

$$I_{C7} \simeq 4 \cdot \underbrace{\frac{1}{1+2/\beta^2}}_{(6.191)} \cdot I_{bias} = 4 \cdot \frac{1}{1+2/50^2} \cdot 0.5mA \simeq 2mA$$

(6.191) → base-current compensation

$$I_{C10} \simeq \frac{1}{1+2/\beta^2} \cdot I_{bias} \simeq 0.5\ mA$$

i) Small-signal voltage gain:



$$I_{E2} = I_{E1} = I_{C7}/2 \simeq 1mA$$

$$I_{E9} \simeq I_{C10} \simeq 0.5mA$$

$$ie_{1,2} = \frac{v_{id}}{2re_1} \longrightarrow ic_2 = \alpha\, ie_{1,2}$$

$$v_{o1} = 2ic_2(ro_2 \parallel ro_4 \parallel R_{in9}) \longrightarrow ic_4 \simeq ic_2$$

$$A_{v1} = \frac{v_{o1}}{v_{id}} \simeq \frac{ro_2 \parallel ro_4 \parallel R_{in9}}{re_1} \qquad (\alpha \simeq 1)$$

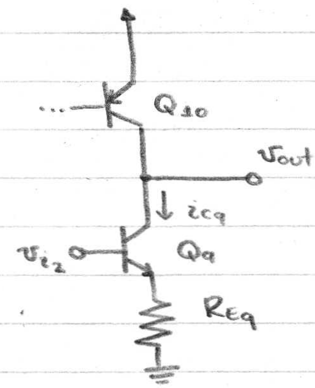
$$re_1 = \frac{V_T}{I_{E1}} = \frac{25m}{1m} = 25\,\Omega \qquad ro_2 = \frac{|V_A|}{I_{C2}} \simeq \frac{80}{1m} = 80\,k\Omega$$

$$re_9 = \frac{V_T}{I_{E9}} = \frac{25m}{0.5m} = 50\,\Omega \qquad ro_4 = \frac{|V_A|}{I_{C4}} \simeq \frac{125}{1m} = 125\,k\Omega$$

$$R_{in9} = (\beta+1)(re_9 + R_{E9})$$
$$= 251 \cdot (50 + 1.4k)$$
$$= 364\,k\Omega$$

$$A_{v1} \simeq \frac{ro_2 \parallel ro_4 \parallel R_{in9}}{re_1}$$
$$= \frac{80k \parallel 125k \parallel 364k}{25}$$
$$= 1.72\ kV/V$$

$$i_{e9} = \frac{v_{i2}}{r_{e9} + R_{E9}} \rightarrow i_{c9} = \alpha i_{e9}$$



$$v_{out} = -i_{c9}(R_{o9} \parallel r_{o10})$$
$$\simeq -i_{c9}(r_{o9} \parallel r_{o10}) \quad \text{(according to prob. stmt.)}$$

$$A_{v2} = \frac{v_{out}}{v_{i2}} \simeq -\frac{r_{o9} \parallel r_{o10}}{r_{e9} + R_{E9}} \quad (\alpha \simeq 1)$$

$$r_{o9} = \frac{|V_A|}{I_{c9}} = \frac{125}{0.5m} = 250\,k\Omega$$

$$r_{o10} = \frac{|V_A|}{I_{c10}} = \frac{80}{0.5m} = 160\,k\Omega$$

$$A_{v2} \simeq -\frac{(r_{o9} \parallel r_{o10})}{r_{e9} + R_{E9}}$$
$$= -\frac{(250k \parallel 160k)}{50 + 1.4k}$$
$$= -67.3\,V/v$$

The voltage gain is then equal to:

$$\frac{v_{out}}{v_{in}} = A_{v1} \cdot A_{v2} = -1.72k \cdot 67.3 = 115.8\,kV/v$$

ii) Input common-mode range:

$$V_{CM_{max}} = V_{CC} - |V_{CEsat_7}| - |V_{BEon_{1,2}}|$$
$$= 3.3 - 0.3 - 0.7$$
$$= 2.3^V$$

$$V_{CM_{min}} = -V_{EE} + |V_{BEon_{3,4}}| + |V_{BEon_5}| + |V_{CEsat_1}| - |V_{BEon_1}|$$
$$= -3.3 + 0.7 + 0.7 + 0.3 - 0.7$$
$$= -2.3^V$$