Solutions to Selected Problems (Problem Set 5)

Chapter 7: *7.1, 7.2, 7.8. 7.9, 7.11, 7.13, 7.16, 7.17, 7.62, 7.63*

7.1

$V_{DD} = V_{SS} = 2.5V$

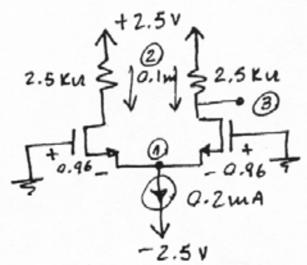
$K_n' \frac{W}{L} = 3 \frac{mA}{V^2}$; $V_{tn} = 0.7V$

$I = 0.2mA$; $R_D = 5K\Omega$

(a) $V_{ov} = \sqrt{I / K_n' W/L} = \sqrt{0.2/3} = \underline{\underline{0.26V}}$

$V_{GS} = V_{ov} + V_t = 0.26 + 0.7 = \underline{\underline{0.96V}}$

(b)



① $V_{S1} = V_{S2} = V_{CM} - V_{GS} = 0 - 0.96 = \underline{\underline{-0.96V}}$

② $I_{D1} = I_{D2} = \frac{I}{2} = 0.1mA$

③ $V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} \times R_D = +2.5 - 0.1 \times 2.5 = \underline{\underline{2.25V}}$

(c) If $v_{CM} = +1V$

$V_{S1} = V_{S2} = +1 - 0.96 = \underline{\underline{0.04V}}$

$I_{D1} = I_{D2} = \underline{\underline{0.1mA}}$

$V_{D1} = V_{D2} = \underline{\underline{2.25V}}$

(d) If $v_{CM} = -1V$

$V_{S1} = V_{S2} = -1 - 0.96 = \underline{\underline{-1.96V}}$

$I_{D1} = I_{D2} = \underline{\underline{0.1mA}}$

$V_{D1} = V_{D2} = \underline{\underline{2.25V}}$

(e) $v_{CMAX} = V_t + V_{DD} - \frac{I}{2} R_D$

$= 0.7 + 2.5 - 0.1 \times 2.5 = \underline{\underline{+2.95V}}$

(f) $v_{CMIN} = -V_{SS} + V_{CS} + V_t + V_{ov}$

$= -2.5 + 0.3 + 0.7 + 0.26$

$= \underline{\underline{-1.24V}}$

$V_{S\,min} = v_{CMmin} - V_{GS}$

$= -1.24 - 0.96 = \underline{\underline{-2.2V}}$

7.2

(a) $V_{ov} = -\sqrt{I / K_p' (W/L)}$

$= -\sqrt{0.7/3.5} = \underline{\underline{-0.45V}}$

$V_{GS} = V_{ov} + V_t = -0.45 - 0.8$

$= \underline{\underline{-1.25V}}$

$v_{S1} = v_{S2} = v_G - V_{GS}$

$= 0 + 1.25 = \underline{\underline{+1.25V}}$

$V_{D1} = V_{D2} = \frac{I}{2} \times R_D - V_{DD}$

$= \frac{0.7}{2} \times 2 - 2.5 = \underline{\underline{-1.8V}}$

(b) For Q_1 and Q_2 to remain in saturation:

$V_{DS} \leq V_{GS} - V_t$

$\rightarrow V_{CM} \geq \left(\frac{I}{2} R_D - V_{DD}\right) + V_t$

$V_{CM\,min} = \frac{0.7}{2} \times 2 - 2.5 - 0.8$

$= \underline{-2.6V}$

To allow sufficient voltage for the current source to operate properly:

$V_{CM} \leq V_{SS} - V_{CS} + (V_t + V_{ov})$

$\rightarrow V_{CM\,max} = 2.5 - 0.5 - 1.25$

$= 0.75V$ //

7.8

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)^2$$

$$\frac{200}{2} = \frac{1}{2}\times 90 \times \frac{100}{1.6}(V_{GS}-0.8)^2$$

$$\Rightarrow V_{GS} = \underline{\underline{1.19\,V}}$$

$$g_m = \frac{2I_D}{V_{GS}-V_t} = \frac{2\times 100}{(1.19-1)} = \underline{\underline{1.06\,\frac{mA}{V}}}$$

$$v_{id}\Big|_{\text{full current switching}} = \sqrt{2}\,(V_{GS}-V_t) = \underline{\underline{0.27\,V}}$$

To double this value, $V_{GS}-V_t$ must be doubled which means that I_D should be quadrupled, i.e. I changed to: $\underline{\underline{800\,\mu A}}$

7.9

$$g_m = \frac{2I_D}{V_{ov}} \rightarrow 1m = \frac{I}{0.2}$$

$$\rightarrow I = \underline{\underline{0.2\,mA}}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{ov}^2$$

$$100 = \frac{1}{2}\times 90 \times \frac{W}{L}\times(0.2)^2$$

$$\Rightarrow \frac{W}{L} = \underline{\underline{55.6}}$$

7.11

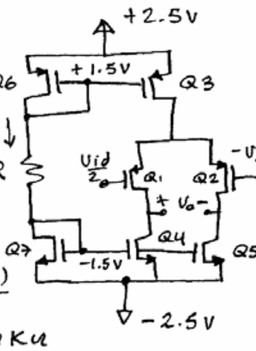
$$V_{ov} = \sqrt{I/k_n'\frac{W}{L}} = \sqrt{\frac{0.5}{0.25\times 50}} = \underline{\underline{0.2\,V}}$$

$$g_m = \frac{I}{V_{ov}} = \frac{0.5\,mA}{0.2\,V} = \underline{\underline{2.5\,\frac{mA}{V}}}$$

$$r_o = \frac{V_A}{I_D} = \frac{10}{(0.5m/2)} = \underline{\underline{40\,k\Omega}}$$

$$A_d = g_m \times (R_D \,||\, r_o)$$
$$= 2.5\,\frac{mA}{V}\,(4\,k\Omega \,||\, 40\,k\Omega)$$
$$= \underline{\underline{9.09\,V/V}}$$

7.13



To obtain $I_{ref} = 100\,\mu A$

$$100\mu = \frac{1.5-(-1.5)}{R}$$

$$\Rightarrow R = \underline{\underline{30\,k\Omega}}$$

$$V_{GS\,7,4,5} = -1.5+2.5 = \underline{\underline{1V}}$$

$$V_{ov\,7,4,5} = V_{GS} - V_{tn} = 1-0.7 = 0.3\,V$$

$$V_{GS\,6,3} = 1.5-2.5 = \underline{\underline{-1V}}$$

$$V_{ov\,6,3} = -1-(-0.7) = -0.3\,V$$

The differential half circuit is an active-loaded common-source amplifier.

thus, for Q_1, Q_4:

$$v_{o+} = \frac{v_{id}}{2}\times g_{m1}(r_{o1}\,||\,r_{o4})$$

For Q_2, Q_5:

$$v_{o-} = -\frac{v_{id}}{2}\times g_{m2}(r_{o2}\,||\,r_{o5})$$

Since $r_{o1} = r_{o2} = r_{o4} = r_{o5} \equiv r_o$

$$v_o^+ - v_o^- = v_{id}\times g_{m1,2}\times\frac{r_o}{2}$$

$$\rightarrow \frac{v_o^+ - v_o^-}{v_{id}} = A_d = g_{m1,2}\times\frac{r_o}{2}$$

$$= \frac{g_{m1,2}}{2}\times\frac{V_{An}}{I_{D1,2}}$$

$$= \frac{1}{2}\times\frac{2I_{D1,2}}{V_{ov}}\times\frac{V_{An}}{I_{D1,2}} = \frac{V_{An}}{V_{ov1,2}}$$

thus:

$$80 = \frac{20}{|V_{ov1,2}|} \rightarrow V_{ov1,2} = -0.25$$

PMOS.

Then $V_{GS1,2} = -0.25 - 0.7$

$= \underline{\underline{-0.95\,V}}$

We have: $I_{D7} = I_{D6} = \underline{\underline{100\,\mu A}}$

If we choose:

$I_{D3} = I_{D6} = 100\,\mu A$ //

then $I_{D1} = I_{D2} = I_{D4} = I_{D5} = \underline{\underline{50\,\mu A}}$

To obtain W/L ratios:

$$I_D = \frac{1}{2}\mu C_{ox}(W/L)\,V_{ov}^2$$

$$\Rightarrow \frac{W}{L} = \frac{2 I_D}{\mu C_{ox}\, V_{ov}^2}$$

where:

$\mu_n C_{ox} = 90\,\mu A/V^2$

$\mu_p C_{ox} = 30\,\mu A/V^2$

For Q_7:

$$\left(\frac{W}{L}\right)_7 = \frac{2 \times 100\mu}{90\mu \times (0.3)^2} = \underline{\underline{24.7}}$$

For Q_4 and Q_5:

$$\left(\frac{W}{L}\right)_{4,5} = \frac{2 \times (100\mu/2)}{90\mu \times (0.3)^2} = \underline{\underline{12.3}}$$

For Q_1 and Q_2:

$$\left(\frac{W}{L}\right)_{1,2} = \frac{2 \times (100\mu/2)}{30\mu \times (0.25)^2} = \underline{\underline{53.3}}$$

For Q_6 and Q_3:

$$\left(\frac{W}{L}\right)_{6,3} = \frac{2 \times 100\mu}{30\mu \times (0.3)^2} = \underline{\underline{74.1}}$$

In summary, the results are:

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	
$\mu_n C_{ox}$	30	30	30	90	90	30	90	$\mu A/V^2$
I_D	50	50	100	50	50	100	100	μA
V_{ov}	-0.25	-0.25	-0.3	0.3	0.3	-0.3	0.3	V
W/L	53.3	53.3	74.1	12.3	12.3	74.1	24.7	
V_{GS}	-0.95	-0.95	-1	1	1	-1	1	

$\boxed{7.16}$

$$V_{ov} = -\sqrt{\frac{I}{K_p' W/L}} = -\sqrt{\frac{0.7\,mA}{3.5\,\frac{mA}{V^2}}}$$

$$= \underline{\underline{-0.45\,V}}$$

$$g_m = \frac{I}{|V_{ov}|} = \frac{0.7\,mA}{0.45\,V} = 1.56\,\frac{mA}{V}$$

$$|A_d| = g_m R_D = 1.56 \times 2 = \underline{\underline{3.12}}\ V/V$$

$$|A_{cm}| = \frac{R_D}{2R_{SS}} \cdot \left(\frac{\Delta R_D}{R_D}\right) = \frac{2}{2 \times 30} \times 0.02$$

$$= \underline{\underline{6.7 \times 10^{-4}}}$$

$$CMRR = \frac{3.12}{6.7 \times 10^{-4}} = 4680 \rightarrow \underline{\underline{73.4}}\,dB$$

$\boxed{7.17}$

(a) $I_{D1} = I_{D2} = \dfrac{1\,mA}{2} = 0.5\,mA$

$$I_D = \frac{1}{2} K_n' \frac{W}{L} \cdot V_{ov}^2$$

$$\Rightarrow 0.5\,m = \frac{1}{2} \times 2.5\,m \times V_{ov}^2$$

$$\longrightarrow V_{ov} = 0.632\,V$$

$V_{ov} = V_{GS} - V_t = V_{GS} - 0.7$

$\longrightarrow V_{GS} = 0.632 + 0.7$

$= 1.332\,V$

To obtain 1 mA over $R_{SS} = 1\,k\Omega$

$V_S = 1\,m \times 1\,K = 1\,V.$

$\longrightarrow V_{CM} = V_S + V_{GS} = 1 + 1.332$

$= \underline{\underline{2.332\,V}}$

(b) $g_m = \dfrac{I}{V_{ov}} = \dfrac{1\,mA}{0.632\,V} = 1.6\,\dfrac{mA}{V}$

Eqn. (7.45): $A_d = g_m \cdot R_D$

for $A_d = 8\,V/V$ $\quad R_D = \dfrac{8}{1.6\,m} = \underline{\underline{5\,k\Omega}}$

(c) At the drains:

$V_{D1} = V_{D2} = 5V - \frac{1mA}{2} \times 5K\Omega$
$= \underline{\underline{+2.5V}}$

(d) Eqn. (7.39):

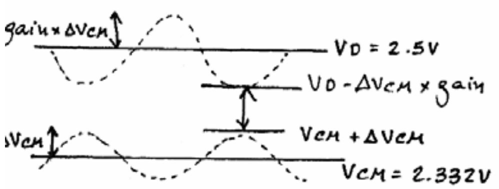
$$\frac{V_{o1}}{V_{icm}} = \frac{-R_D}{\frac{1}{g_m} + 2R_{SS}}$$

$$\Rightarrow |A_{cm}| = \left|\frac{\Delta V_{D1}}{\Delta V_{cm}}\right| = \frac{5K}{\frac{1}{1.6m} + 2\times 1K}$$

$= \underline{\underline{1.9}}$ V/V

(e) On the edge of the triode region:

$V_G - V_D = V_t$



f: $V_G - V_D = V_t$

$\Rightarrow V_{CM} + \Delta V_{CM} - V_D + \Delta V_{CM}|A_{cm}| = V_t$.

$\Rightarrow 2.332 + \Delta V_{CM} - 2.5 + \Delta V_{CM} \cdot 1.9 = 0.7$

$2.9\,\Delta V_{CM} = 0.868$

$\Delta V_{CM} = \underline{\underline{0.3V}}$

7.62

For each transistor $I_D = I/2$.

From Eqn. (7.147): $r_{o2} = r_{o4} = r_o$

$A_d = \frac{1}{2} g_m r_o$

but $g_m = \frac{2I_D}{V_{OV}}$ and $r_o = \frac{V_A}{I_D}$

$$\Rightarrow A_d = \frac{1}{2}\left(\frac{2I_D}{V_{OV}}\right)\frac{V_A}{I_D} = \frac{V_A}{V_{OV}}$$

$\rightarrow 80$ V/V $= 20V/V_{OV}$

$\rightarrow V_{OV} = 20/80 = 0.25V$

Finally,

$I = 2I_D = K\frac{W}{L} V_{OV}^2$
$= 3.2\frac{mA}{V^2}(0.25V)^2 = \underline{\underline{0.2mA}}$

7.63

For all transistors $I_D = I/2$ and all r_o's are equal.

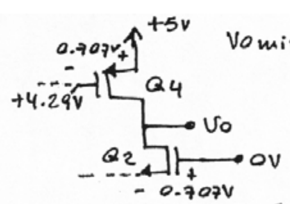
$$V_{OV_{100}} = \sqrt{\frac{2I_D}{K'W/L}} = \sqrt{\frac{100\mu A}{0.2mA/V^2}} = 0.707V$$

$$V_{OV_{200}} = \sqrt{\frac{400\mu A}{0.2mA/V^2}} = 1.414V$$

(a) For $I = 100\mu A$:

Range of the differential mode is $-\sqrt{2}V_{OV} \le v_{id} \le \sqrt{2}V_{OV}$ (as in Eqn. (7.10))

But the range of v_o is limited by the requirement of keeping the transistors in saturation mode.



$V_{o\,min} = V_{G2} - V_t$
$= 0 - V_t$
$= \underline{-V_t}$

$V_{o\,max} = V_{G4} + |V_t|$
$= V_{DD} - |V_{GS}| + |V_t|$
$= 5 - 0.707 = \underline{\underline{4.29V}}$

$g_m = \frac{2I_D}{V_{OV}} \rightarrow g_{m1} = g_{m2} = \frac{100\mu}{0.707}$
$= \underline{0.1414mA/V}$

$r_o = \frac{V_A}{I_D} \Rightarrow r_{o2} = r_{o4} = \frac{20}{50\mu} = \underline{\underline{400K\Omega}}$

$R_o = r_{o2} \| r_{o4} = \frac{1}{2} \times 400K = \underline{200K\Omega}$

$A_d = \frac{1}{2} g_m r_o = \frac{1}{2} \times (0.1414m)(400K)$

$Ad = \underline{\underline{28.28}}\ V/V$

(b) For $I = 400\mu A$:
Linear range of V_O

$V_{O\,min} = -V_t$ //
$V_{O\,max} = (5V - |V_{GS}|) + |V_t|$
$= \underline{\underline{5 - 1 = 4V}}$

$g_m = \frac{2 \times 200\mu}{1.414} = \underline{\underline{0.28}}\ mA/V$

$r_o = 20/(400/2) = \underline{\underline{100K\Omega}}$

$R_o = 1/2\ 100K\Omega = \underline{\underline{50}}\ K\Omega$

$Ad = \frac{1}{2}(0.28m \times 100K) = \underline{\underline{14}}\ V/V$