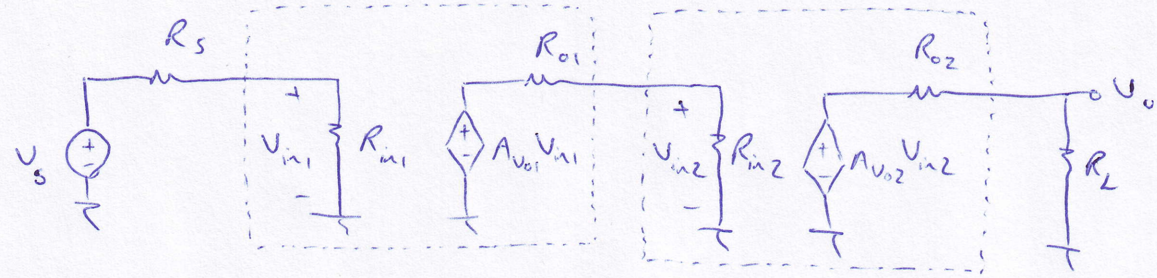
Question 1)

a)



$$\frac{V_{in1}}{V_s} = \frac{R_{in1}}{R_{in1}+R_s}$$

$$\frac{V_{in2}}{A_{vo1}V_{in1}} = \frac{R_{in2}}{R_{in2}+R_{o1}}$$

$$\frac{V_o}{A_{vo2}V_{in2}} = \frac{R_L}{R_L+R_{o2}}$$

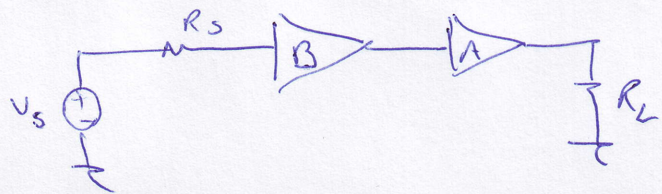
$$\Rightarrow \frac{V_o}{V_s} = \frac{R_{in1}}{R_{in1}+R_s} A_{vo1} \frac{R_{in2}}{R_{in2}+R_{o1}} A_{vo2} \frac{R_L}{R_L+R_{o2}}$$

1. It is better to have a first stage with a large input impedance to maximize $\frac{R_{in1}}{R_{in1}+R_s}$

2. It is better to have a last stage with a small output impedance to maximize $\frac{R_L}{R_L+R_{o2}}$

3. It is desirable to have a second stage with an input impedance bigger than the output impedance of the first stage (to have a large $\frac{R_{in2}}{R_{in2}+R_{o1}}$)

$\Rightarrow$ Amplifier B seems to be a better choice for the first stage.



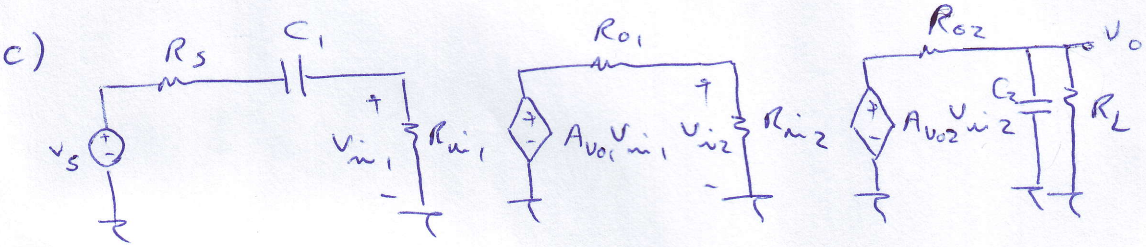
b) $$\frac{V_o}{V_s} = \frac{R_{in1}}{R_{in1}+R_s} \times A_{vo1} \times \frac{R_{in2}}{R_{in2}+R_{o1}} \times A_{vo2} \times \frac{R_L}{R_L+R_{o2}}$$

$$= \frac{100^k}{100^k+10^k} \times 100 \times \frac{2^k}{2^k+1^k} \times 50 \times \frac{100}{100+100} = \boxed{1515 \ V/V}$$

if amplifier # 1 had been chosen for the first stage :

$$\frac{V_o}{V_s} = \frac{2^k}{2^k+10^k} \times 50 \times \frac{100^k}{100^k+0.1^k} \times 100 \times \frac{0.1^k}{0.1^k+1^k} = 75.7 \ V/V \ll 1515 \ V/V$$

c)



$$\frac{V_{in1}}{V_s} = \frac{R_{in1}}{R_{in1}+R_s+\frac{1}{sC_1}} = \frac{R_{in1}}{R_{in1}+R_s} \times \frac{s}{s+\frac{1}{C_1(R_{in1}+R_s)}}$$

$$\frac{V_{i2}}{V_{i1}} = A_{vo1}\frac{R_{in2}}{R_{in2}+R_{o1}}$$

$$\frac{V_o}{V_{i2}} = A_{vo2}\frac{R_L \| \frac{1}{C_2 s}}{R_L \| \frac{1}{C_2 s}+R_{o2}} = A_{vo2}\frac{\frac{R_L}{R_L C_2 s+1}}{\frac{R_L}{R_L C_2 s+1}+R_{o2}} = A_{vo2}\frac{R_L}{R_L+R_{o2}} \times \frac{1}{1+s\frac{R_L R_{o2}}{R_L+R_{o2}}C_2}$$

$$\Rightarrow \frac{V_o}{V_s} = \underbrace{\frac{R_{in1}}{R_{in1}+R_s} \times A_{vo1} \times \frac{R_{in2}}{R_{in2}+R_{o1}} \times A_{vo2} \times \frac{R_L}{R_L+R_{o2}}}_{\text{frequency independent}} \times \underbrace{\frac{s}{s+\frac{1}{C_1(R_{in1}+R_s)}}}_{\text{high pass}} \times \underbrace{\frac{1}{1+s\, R_L \| R_{o2} C_2}}_{\text{low pass}}$$

$$\omega_{o1} = \frac{1}{C_1(R_{in1}+R_s)} \Rightarrow f_{o1} = \frac{1}{2\pi C_1(R_{in1}+R_s)} \simeq 289 \ Hz$$

$$\omega_{o2} = \frac{1}{C_2(R_L \| R_{o2})} \Rightarrow f_{o2} = \frac{1}{2\pi C_2(R_L \| R_{o2})} \simeq 3.18 \ MHz$$

by inspection :

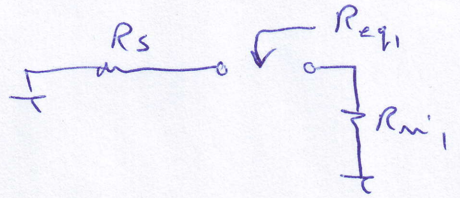
$s = 0 \Rightarrow C_1 \to$ O.C. $\Rightarrow V_{in1} = 0$

$s = \infty \Rightarrow C_1 \to$ S.C. $\Rightarrow \dfrac{V_{in1}}{V_s} = \dfrac{R_{in1}}{R_{in1} + R_s}$

$\Rightarrow$ first stage is higpass

$$K = \frac{R_{in1}}{R_{in1} + R_s}$$

ω_{01} : $\to$ kill V_s :



$R_{eq1} = R_{in1} + R_s$

$$\Rightarrow \omega_{01} = \frac{1}{C_1(R_s + R_{in1})} \Rightarrow f_{01} = 289^{Hz}$$

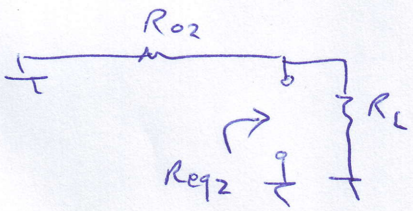
$s = 0 \Rightarrow C_2 \to$ O.C. $\Rightarrow \dfrac{V_o}{V_{in2}} = A_{vo2}\dfrac{R_L}{R_L + R_{o2}}$

$s = \infty \Rightarrow C_2 \to$ S.C. $\Rightarrow V_o = 0$

$\Rightarrow$ last stage is lowpass

$$K = A_{vo2}\frac{R_L}{R_L + R_{o2}}$$

ω_{02} : $V_s \to 0$:



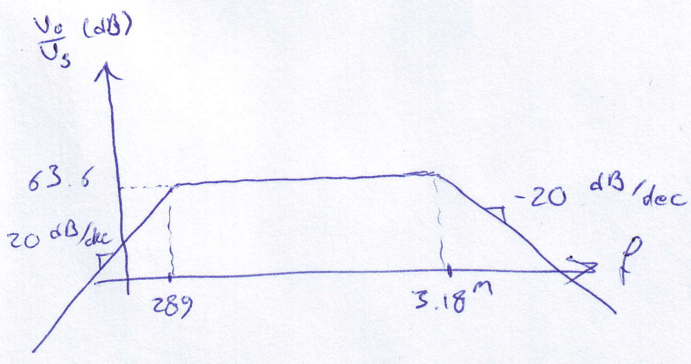
$R_{eq2} = R_L \parallel R_{o2}$

$$\Rightarrow \omega_{02} = \frac{1}{C_2 R_{eq2}} \Rightarrow f_{02} = 3.18^{MHz}$$

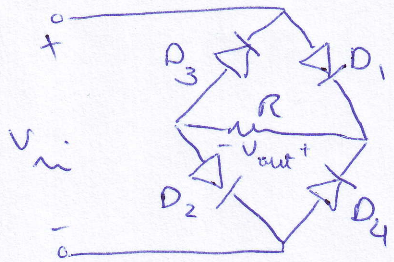
for $289^{Hz} \le f \le 3.18^{MHz}$ (midband frequencies) :

$$A_v \simeq \frac{R_{in1}}{R_{in1} + R_s} \times A_{vo1} \times \frac{R_{in2}}{R_{in2} + R_{o1}} \times A_{vo2} \times \frac{R_L}{R_L + R_{o2}} = 1515 = 63.6^{dB}$$

the amplifier is bandpass.



Question 2)



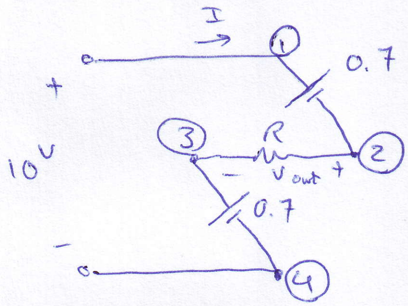
$n = 2$

$V_{BE(on)} = 0.7^V$

$R = 4^{k\Omega}$

a) $V_{in} = 10^V$

assume D_1 and D_2 are "on", D_3 and D_4 are "off"



$V_{in} = 0.7 + V_{out} + 0.7$

$\Rightarrow V_{out} = V_{in} - 1.4$

$\Rightarrow \boxed{V_{out} = 8.6^V}$

verify assumptions :

$I = \frac{8.6}{4} = 2.15^{mA}$

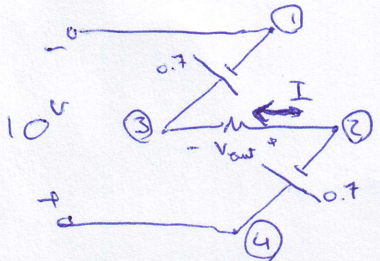
$I_{D_1} = I_{D_2} = I > 0$ ✓

$V_{D_3} = V_3 - V_1 = -0.7 - V_{out} = -9.3^V < 0.7^V$ ✓

$V_{D_4} = V_4 - V_2 = -0.7 - V_{out} = -9.3^V < 0.7^V$ ✓

b) $V_{in} = -10^V$

assume D_3 and D_4 are "on", D_1 and D_2 are off"



$-10^V = -0.7 - V_{out} - 0.7$

$\Rightarrow \boxed{V_{out} = 8.6^V}$ $\Rightarrow I = \frac{8.6}{4} = 2.15^{mA}$

verify assumptions :

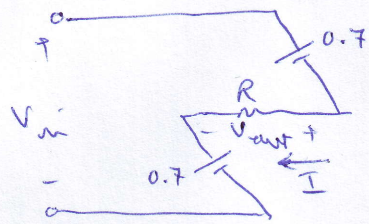
$I_{D_3} = I_{D_4} = I > 0$ ✓

$V_{D_1} = V_1 - V_2 = -0.7 - V_{out} = -8.6 - 0.7 = -9.3^V < 0.7^V$ ✓

$V_{D_2} = V_3 - V_4 = -V_{out} - 0.7 = -8.6 - 0.7 = -9.3^V < 0.7^V$ ✓

c) $V_{in} \geq 2 \times 0.7 = 1.4^V$: D_1 and D_2 are "on"

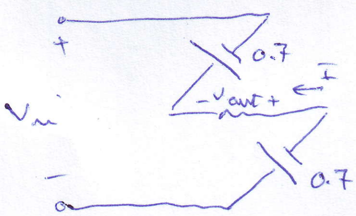
D_3 and D_4 are "off"



$$\left.\begin{aligned} I &= \frac{V_{in} - 2\times 0.7}{R} \\ V_{out} &= RI \end{aligned}\right\} \Rightarrow \boxed{V_{out} = V_{in} - 1.4^V}$$

$V_{in} \leq -2\times 0.7 = 1.4^V$: D_3 and D_4 are "on"

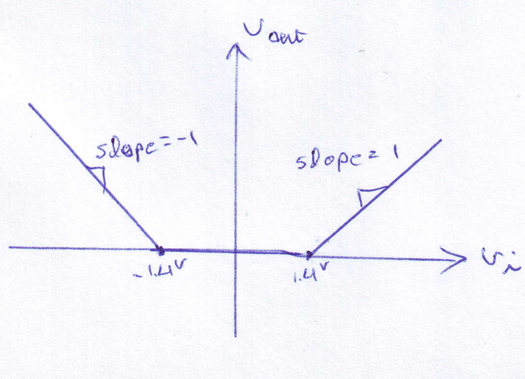
D_1 and D_2 are "off"



$$\left.\begin{aligned} I &= \frac{-V_{in} + 2\times 0.7}{R} \\ V_{out} &= RI \end{aligned}\right\} \Rightarrow \boxed{V_{out} = -V_{in} + 1.4^V}$$

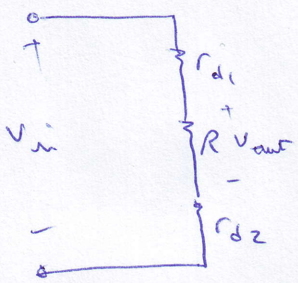
$-1.4^V \leq V_{in} \leq 1.4^V$: all diodes are "off"

$\Rightarrow I = 0 \Rightarrow \boxed{V_{out} = 0}$



d) $I = 2.15^{mA}$

$$\Rightarrow \quad r_{d_1} = r_{d_2} = \frac{nV_T}{I} = \frac{50^{mV}}{2.15^{mA}} = 23.256^{\Omega}$$



$$V_{out}(dc) = 8.6^{V}$$

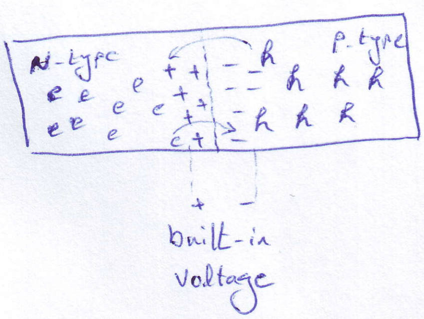
$$V_{out}(ac) = \frac{R}{R + r_{d_1} + r_{d_2}} V_{in}(ac)$$

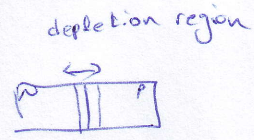
$$= 0.9885\, V_{in}(ac) \quad \Rightarrow \quad A_v = 0.9885^{V/V}$$

$$\Rightarrow \quad V_{out} = 8.6^{V} + 0.9885 \cos 10t$$

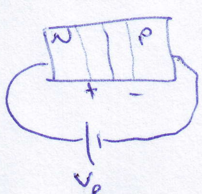
Question 3)

a) At room temperature N-type silicon has extra free electrons and P-type silicon has free holes. When a PN-junction is formed according to the thermodynamic law of maximum entropy, free electrons from the N-type material diffuse to the p-type material (due to lower concentration of free electrons in this region) and recombine with the majority carriers of the p-type material (holes). In the same manner free holes from the p-type material diffuse to the N-type material and recombine withe the free electrons of the N-type material. This creates local charge sites that do not move and create an internal electric field. This field is the source of drift for any free carriers that diffuse into the depletion region. When the rate of diffusion equals the rate of drift a steady-state condition is obtained and no more macroscopic changes occur.

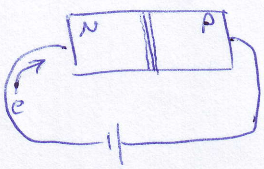




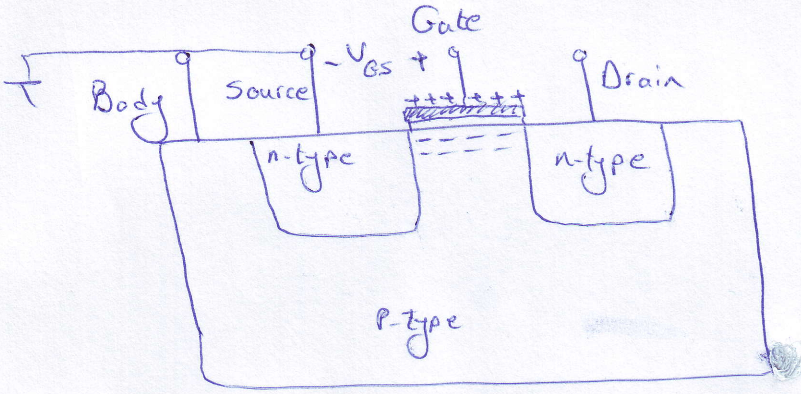
ii) When a reverse-bias voltage is applied to junction, depletion-region widens to accomodate for higher reverse-bias.



ii) Forward-bias voltage injects majority carrier electrons into n-type, majority carrier holes into p-type material. Dominant current is diffusion current. Diffusion of majority carriers across the junction and the subsequent recombination completes the circuit. The process "takes-off" after 0.7^v and collapses the built-in voltage to almost zero.

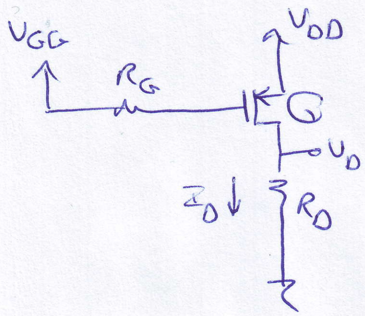


b)



When a positive voltage is applied to the gate, electrons (minority carriers of the p-type substrate) are absorbes to the area right below the gate. If V_{GS} is bigger than the threshold voltage (V_t), the inversion layer is big enough to form a channel between drain and source.

Question 4)



$V_{DD} = 5^V$, $V_{GG} = 3^V$, $R_G = 4^{k\Omega}$, $R_D = 2^{k\Omega}$

$\mu_p C_{ox} = 100^{\mu A/V^2}$, $|V_{tp}| = 1^V$, $\lambda = 0.025^{1/V}$, $\frac{W}{L} = 20$

assume that Q is in saturation :

$$\left.\begin{aligned} & I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{tp}|)^2(1 + \lambda V_{SD}) \\ & V_S = V_{DD} = 5^V \\ & I_G = 0 \Rightarrow V_G = V_{GG} = 3^V \\ & V_D = R_D I_D \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow I_D = \frac{1}{2} \times 100^{\mu} \times 20 \times (5-3-1)^2 (1 + 0.025(5 - 2^k I_D))$$

$$\Rightarrow I_D = 1000^{\mu} (1)^2 \left(1 + \frac{5 - 2^k I_D}{40}\right)$$

$$I_D = 1.125^m - \frac{2}{40} I_D \Rightarrow \boxed{I_D = 1.07^{mA}}$$

$$\Rightarrow \boxed{V_D = R_D I_D = 2.14^V}$$

assumption verification :

$V_{SG} = 5 - 3 = 2^V > |V_{tp}| = 1^V$ ✓

$V_{SD} = 5 - 2.14 = 2.86^V > V_{SG} - |V_t| = 5 - 3 - 1 = 1^V$ ✓