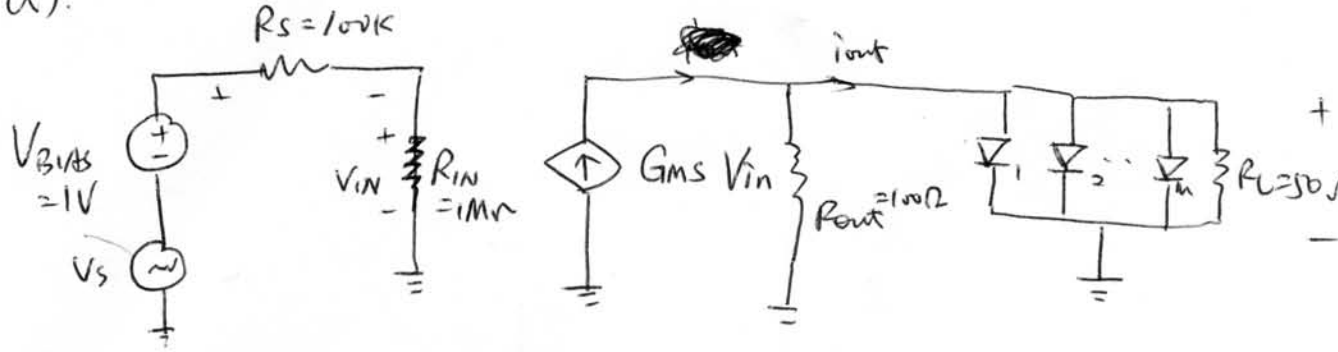
Solutions 2003

Q 1.

a).



For all the diodes to be conduct, assuming CVDM.

$V_{out} = 0.7V.$

$$V_{IN} = \frac{R_{IN}}{R_S + R_{IN}} \cdot V_{BIAS} = 0.91V.$$

Assuming on the conducting Bondary. $I_{DM} = 0.$

So:

$$G_{MS} V_{in} = \frac{0.7}{R_{out}} + \frac{0.7}{R_L}$$

$$G_{MS} \times 0.91 = \frac{0.7}{100} + \frac{0.7}{50} \Rightarrow G_{MS} = 23.1 mA/V.$$

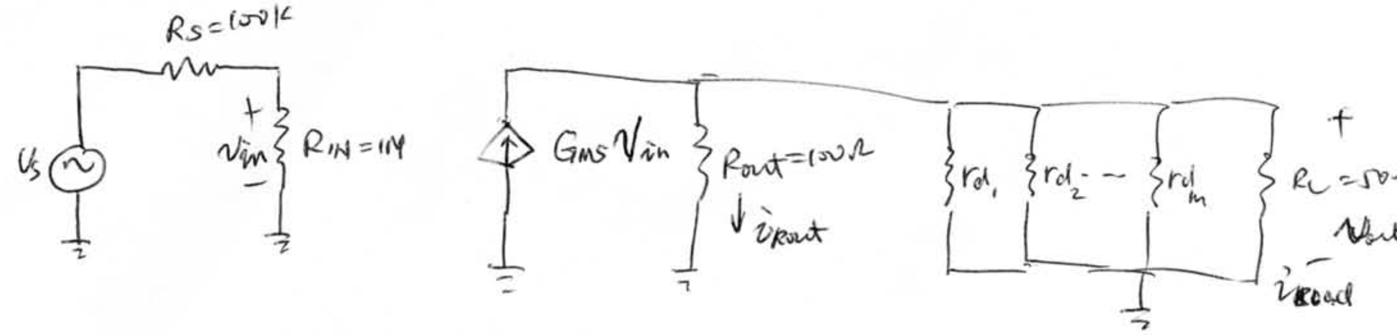
b). $G_{MS} = 50 mA/V.$ assuming CVDM.

$$G_{MS} \times V_{IN} = \frac{0.7}{100} + \frac{0.7}{50} + I_D \times M.$$

$$50 \times 10^{-3} \times 0.91 = 0.7 \times \frac{3}{100} + I_D \times M$$

$$I_D = \frac{0.0245}{M} = \frac{24.5 mA}{M}$$

c).



$$r_d = \frac{n \cdot V_T}{I_D}.$$

$$I_D = \frac{24.5mA}{M} \qquad r_d = \frac{2 \times 25mV}{24.5mA} \cdot M = 2.04\,M$$

d). $$V_{IN} = \frac{R_{IN}}{R_{IN}+R_S} \cdot V_s = 0.91\,|V_s|\,Sin(\omega t).$$

$$G_{MS}V_{in} = 50\times10^{-3} \times 0.91\,|V_s|\,Sin(\omega t) = 0.0455\,|V_s|\,Sin(\omega t)$$

$$= 45.5\,|V_s|\,Sin(\omega t)\ mA.$$

Small signal output

$$V_{out} = R_L \times 100\,Sin(\omega t)\ \mu A = 5\,Sin(\omega t)\ mV$$

$$i_{Rout} = \frac{V_{out}}{R_{out}} = \frac{5\,Sin(\omega t)}{100} = 50\,Sin(\omega t)\ \mu A.$$

$$i_{load} = 100\,Sin(\omega t)\ \mu A.$$

$$M\,i_d = G_{MS}V_{in} - i_{Rout} - i_{Load}.$$

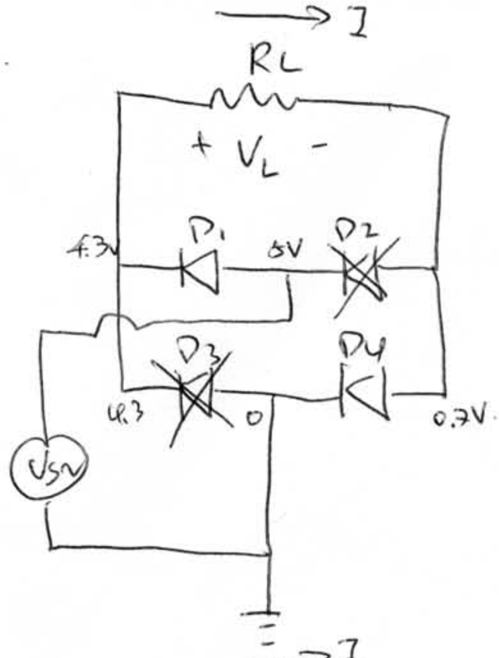
$$i_d = \frac{45.5\times10^3\,|V_s|\,sin(\omega t) - 150\,Sin(\omega t)}{M}.$$

$$i_d = \frac{V_{out}}{r_d} = \frac{5\,Sin(\omega t)\times10^3}{2.04\,M} = \frac{2451}{M}\,Sin(\omega t)\ \mu A.$$

$$45.5\times10^3\,|V_s| - 150 = 2451$$

$$|V_s| = 0.057\,V = 57\,mV.$$

Q2.



CVDM n=1.

a). D1, D4 is ON

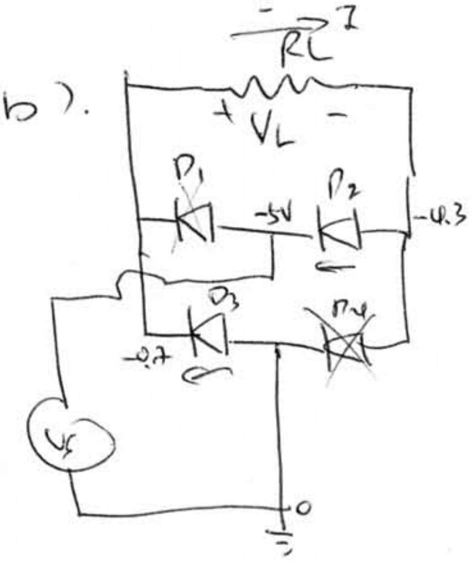
D2, D3 off. Vs=5V.

$I = \frac{4.3-0.7}{R_L} = \frac{3.6}{R_L}$ $V_{D2} = -4.3V$ $V_{D3} = -4.3V$

D2. D3 are reversely biased.

$V_L = IR_L = 3.6V$.

b).



Vs=-5V

D2, D3 ON

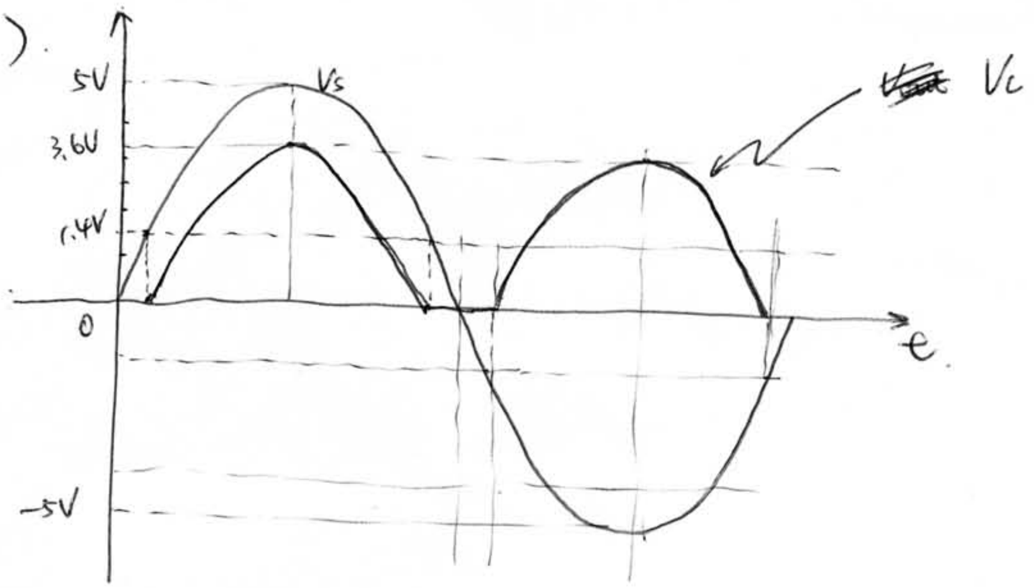
D1, D4 off

$I = \frac{-4.3+0.7}{R_L}$

$V_{D1} = -4.3V$
$V_{D4} = -4.3V$ > Reversely biased.

$V_L = 3.6V$.

c).



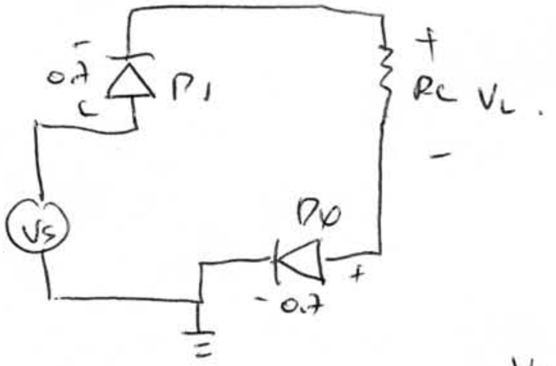
Q2. d). The swing is from -10mV to 10mV. No diodes can be on. $V_L = 0$ always.

e). $V_S = 5V + 10\sin(\omega t)\, mV$.

The DC BIAS is 5V.

So. D2 D3 off. D1 D4 ON.

DC current CVDM



$$I_D = \frac{5-1.4}{R_L} = \frac{3.6}{R_L} \qquad V_L = 3.6V$$

$$r_d = \frac{nV_T}{I_D} = \frac{1 \times 25mV}{3.6} \times R_L = 7 \times 10^{-3} R_L$$

$$V_\ell = \frac{R_L}{R_L + 2r_d} \times V_s$$

$$V_L = 3.6V + \frac{R_L}{R_L + 2r_d} \times 10\sin(\omega t)\, mV.$$

$$= 3.6V + \frac{1}{1 + 1.4 \times 10^{-2}} \times 10\sin(\omega t)\, mV$$

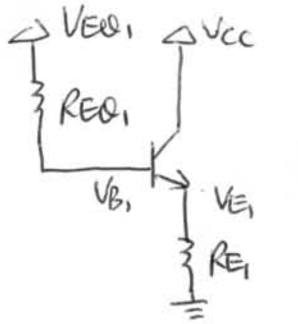
$$= 3.6V + 9.862\sin(\omega t)\, mV.$$

Final 2003 Fall, Solutions Q3, Q7.

Q3

a) $V_{EQ_1} = \frac{R_{B2}}{R_{B2}+R_{B1}} \times V_{cc} = \frac{40K}{40K+120K} \times 12 = 3V$

$R_{EQ_1} = R_{B1} \| R_{B2} = 120K \| 40K = 30K\Omega$



$$\begin{cases} I_{B_1} = \frac{V_{EQ_1} - V_{B_1}}{R_{EQ_1}} \\ I_{E_1} = \frac{V_{E_1}}{R_{E_1}} \\ I_{E_1} = (\beta+1) I_{B_1} \\ V_{B_1} = V_{E_1} + 0.7 \end{cases} \Rightarrow \frac{V_{E_1}}{R_{E_1}} = (\beta+1)\frac{V_{EQ_1} - V_{E_1} - 0.7}{R_{EQ_1}} \quad (*)$$

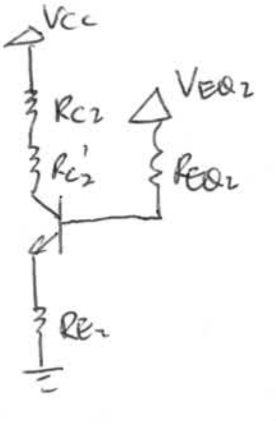
$$\frac{V_{E_1}}{3K} = 50 \times \frac{3 - 0.7 - V_{E_1}}{30K} \qquad \boxed{V_{E_1} = 1.92V.}$$

$$\boxed{I_{C_1} = \frac{\beta}{\beta+1} I_{E_1} = \frac{49}{50} \times \frac{1.92V}{3K\Omega} = 0.627\,mA}$$

For Q2.

$V_{EQ_2} = \frac{R_{B4}}{R_{B3}+R_{B4}} \times V_{cc} = \frac{20K}{120K+20K} \times 12 = 1.714V$

$R_{EQ_2} = R_{B3} \| R_{B4} = 17.14K\Omega.$



Use (*)

$$\frac{V_{E_2}}{1K} = 50 \times \frac{1.714 - V_{E_2} - 0.7}{17.14K}$$

$$\boxed{V_{E_2} = 0.755V.}$$

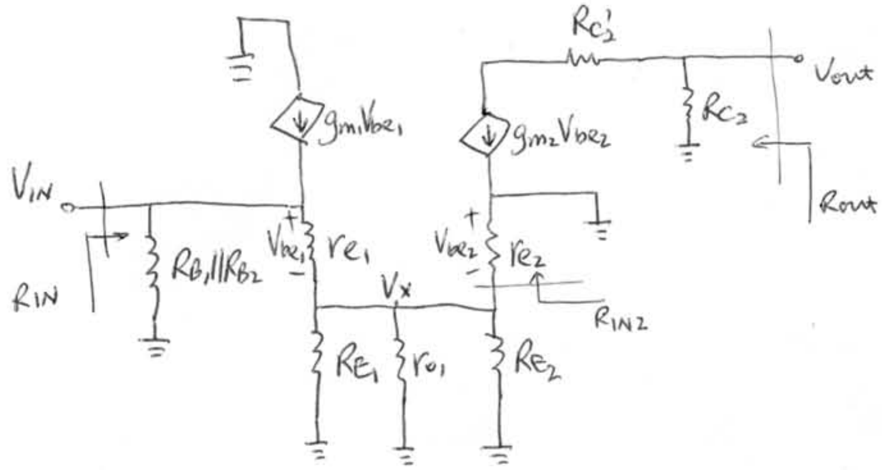
$$\boxed{I_{C_2} = \frac{\beta}{\beta+1} I_{E_2} = \frac{49}{50} \times \frac{0.755}{1K} = 0.74\,mA}$$

No other solutions, must be the same as the value specified here.

Q3

b)

Without R_{C2}', take (−0.25)



c)

$$R_{IN} = R_{B1}||R_{B2} \,||\, (\beta+1)\left[r_{e1} + R_{E1}||r_{o1}||R_{E2}||r_{e2}\right]$$

d) $R_{IN2} = r_{e2}$

e) $R_{out} = R_{C2}$

f).

$$V_{out} = -g_{m2}V_{be2}(R_{C2}||R_L)$$

$$V_{be2} = -V_x$$

$$V_x = V_{in} \cdot \frac{R_{E2}||R_{E1}||r_{e2}}{r_{e1} + R_{E1}||R_{E2}||r_{e2}}$$

$$g_m = \frac{I_C}{V_T} = 0.032\ A/V$$

$$r_e = \frac{\alpha}{g_m} = 30.625\Omega$$

neglect Early effect.

$$\frac{V_{out}}{V_{IN}} = \frac{g_{m2}\,(R_{C2}||R_L) \cdot R_{E2}||R_{E1}||r_{e2}}{r_{e1} + R_{E1}||R_{E2}||r_{e2}}$$ //

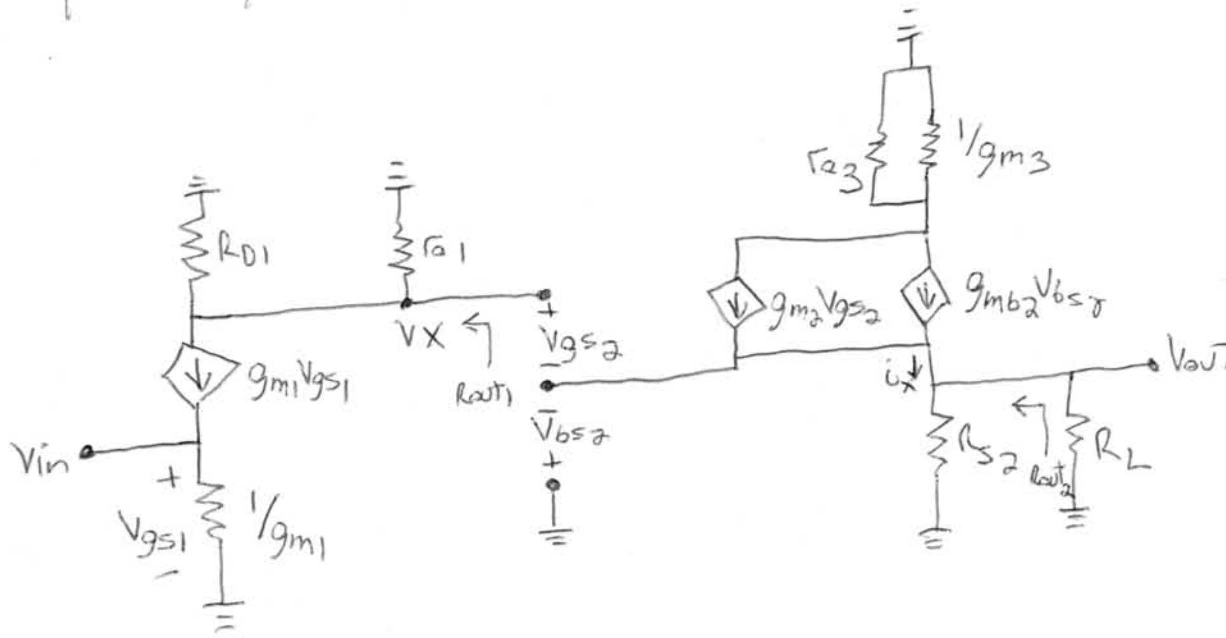
if R_L is not connected. open circuit gain is

$$A_v = \frac{0.032 \times 10K \times 3K||1K||30.625}{30.625 + 3K||1K||30.625} = 156.8\ V/V$$

Q4

	Body Eff.	CLM
M1	N	Y
M2	Y	N
M3	N	Y

a)



b) $$\boxed{\frac{V_X}{V_{in}} = -g_m\left(r_{o1} \,//\, R_{D1}\right)}$$

c) $V_X = V_{gs2} + V_{out}$; $V_{gs2} = V_X - V_{OUT}$

$V_{BS2} = -V_{out}$

① $i_X = g_{m2}V_{gs2} - g_{mb2}V_{out}$ $\Rightarrow$ ② $i_X = \dfrac{V_{OUT}}{R_{s2}//R_L}$

$$\Rightarrow \frac{V_{out}}{R_{s2}//R_L} = g_{m2}V_{gs2} - g_{mb2}V_{out}$$

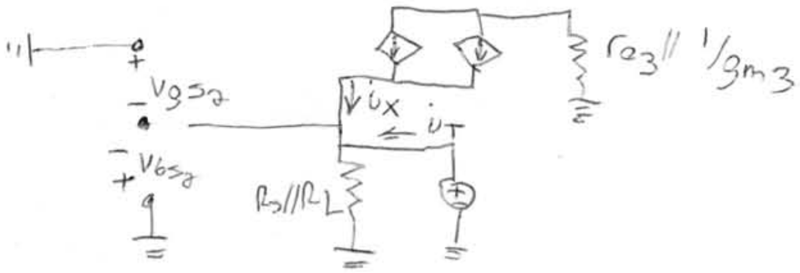
$$\Rightarrow \frac{V_{out}}{R_{s2}//R_L} + g_{mb2}V_{out} = g_{m2}(V_X - V_{OUT})$$

$$\Rightarrow \frac{V_{out}}{R_{s2}//R_L} + g_{mb2}V_{out} + g_{m2}V_{OUT} = g_{m2}V_X \Rightarrow \boxed{\frac{V_{out}}{V_X} = \frac{g_{m2}}{\frac{1}{R_{s2}//R_L} + g_{m2} + g_m}}$$

d) $\frac{V_{out}}{V_{in}} = \frac{V_x}{V_{in}} \cdot \frac{V_{out}}{V_x} = \left[-g_{m1}(r_{o1} // R_{D1})\right] \left[\frac{g_{m2}}{\frac{1}{R_{s2}//R_L} + g_{m2} + g_{mb2}}\right]$

e) $R_{out1} = r_{o1} // R_{D1}$

f)



$\frac{V_{out}}{R_s//R_L} = i_T + i_x$; $V_x = 0V$ $\quad V_{gs} = -V_{out}$

$= i_T + g_{m2}V_{gs2} + g_{mb2}V_{bs2}$

$= i_T + g_{m2}V_{gs2} - g_{mb2}V_{out}$

$\frac{V_{out}}{R_s//R_L} + g_{mb2}V_{out} = i_T - g_{m2}V_{out} \Rightarrow i_T = \frac{V_{out}}{R_s//R_L} + g_{mb2}V_{out} + g_{m2}V_{out}$

$\Rightarrow R_{out} = \frac{V_{out}}{i_T} = \boxed{\frac{1}{\frac{1}{R_s//R_L} + g_{m2} + g_{mb2}}}$

Q#5

a) BOUNDARY: $V_{BE} = 0.7$; $V_B = V_C$

(3 PTS)

$$V_{B_1} = V_{B_2} = 1.8V$$
$$V_{C_1} = V_{C_2} = 1.8V$$

$$I_{C_1} = I_{C_2} = \frac{V_{CC} - V_{C_1}}{R_{C_1}} = \frac{3.3 - 1.8}{R_{C_1}} = \frac{1.5}{R_{C_1}}$$

SINCE $V_{in} = 0$ $\quad V_{E_1} = V_{E_2} \rightarrow$ NO DC CURRENT THROUGH R_E

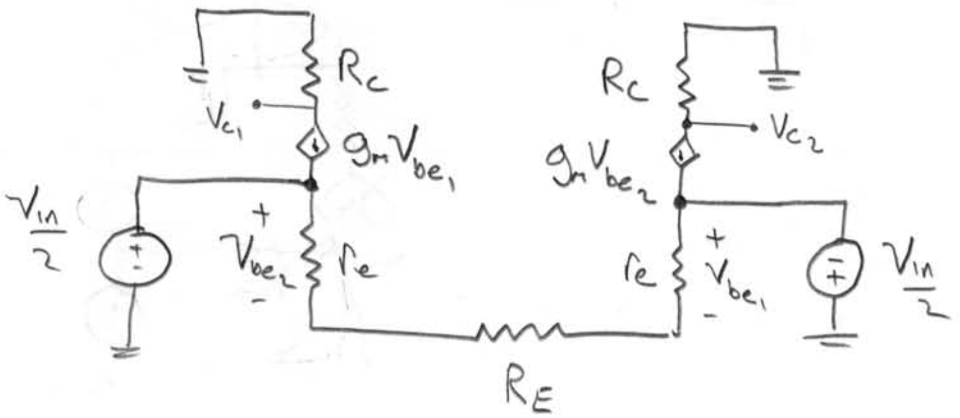
$$I_{E_1} = I_{E_2} = 1\,mA$$

$$I_{C_1} = \alpha I_{E_1} = \frac{99}{100} \times 1\,mA = .99\,mA$$

$$R_{C_1} = R_{C_2} = \frac{1.5V}{0.99} = 1.515\,K\Omega$$

b)

(2 PTS)



c)

(2 PTS)

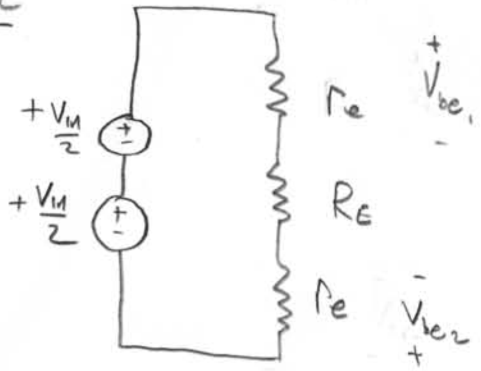
$$V_{C_1} - V_{C_2} = -g_m R_C (V_{be_1} - V_{be_2})$$

KVL: $\frac{V_{in}}{2} - V_{be_1} - V_{RE} + V_{be_2} + \frac{V_{in}}{2} = 0$

$$V_{in} - V_{RE} = V_{be_1} - V_{be_2}$$

VOLATAGE DIVIDER

$$V_{RE} = V_{in}\left(\frac{R_E}{2r_e + R_E}\right)$$



$$V_{be_1} - V_{be_2} = V_{in} - V_{RE}$$

$$= V_{in}\left(1 - \frac{R_E}{2r_e + R_E}\right) = V_{in}\left[\frac{2r_e}{2r_e + R_E}\right]$$

$$\boxed{\frac{V_{C_1} - V_{C_2}}{V_{in}} = -g_m R_C\left(\frac{2r_e}{2r_e + R_E}\right)} \qquad \boxed{= \frac{-g_m R_C\, 2r_\pi}{2r_\pi + (\beta+1)R_E}}$$

d) (3PTS)

$$R_{in} \equiv \frac{V_{in}}{i_b} = \frac{V_{in}}{-g_m V_{be} + i_e} = \frac{V_{in}}{i_e(1 - g_m r_e)}$$

$$i_e = \frac{V_{RE}}{R_E} = \frac{1}{R_E}\left[V_{in}\left(\frac{R_E}{2r_e + R_E}\right)\right] = \frac{V_{in}}{2r_e + R_E}$$

$$R_{in} = \frac{\left(\frac{V_{in}}{1 - g_m r_e}\right)}{\frac{V_{in}}{2r_e + R_E}} = \frac{2r_e + R_E}{1 - g_m r_e} = (\beta+1)(2r_e + R_E)$$

$$\boxed{R_{in} = (\beta+1)(2r_e + R_E)}$$

$$\boxed{R_{in} = 2r_\pi + (\beta+1)R_E}$$

Q6

a) $I_{M_1} = \frac{1}{2} K_n' \frac{3W}{L} (V_{GS_1} - V_t)^2 = 3 I_{M_2}$

$I_{C1} = I_{M_1} = 3 I_{M2}$

$I_{B1} = \frac{I_{C1}}{\beta} = \frac{3 I_{M2}}{\beta}$

$I_{E1} = \frac{I_{C1}}{\beta}(\beta+1) = \frac{3(\beta+1)}{\beta} I_{M2}$

$I_{M3} = I_{M2}$

b) $V_{out} = V_{CC} - R_D(I_{M1} + I_{M2})$

$= 10 - 1k\Omega(3I_{M2} + I_{M2})$

$= 10 - 4k\Omega \cdot I_{M2}$

c) $V_{out} = 10 - 4k\Omega \cdot 1mA$

$= 6V$

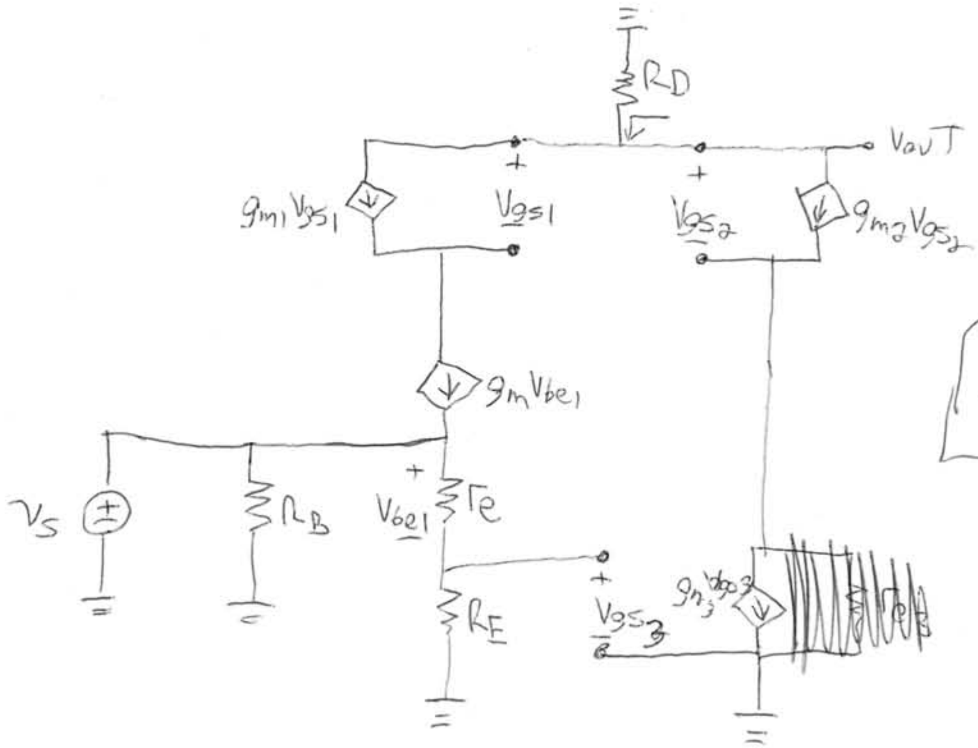
$I_{M2} = I_{M3} \Rightarrow \frac{1}{2} K_n' \frac{W}{L} (6 - V_{S2} - 1)^2 = \frac{1}{2} K_n' \frac{W}{L} (3.03 - 1)^2 (1 + \frac{1}{4} V_S$

$25 - 10V_{S_2} + V_{S_2}^2 = 4.122 + 1.0305 V_{S2}$

$\Rightarrow V_{S2}^2 - 11.03 V_{S2} + 20.878 = 0$

$\Rightarrow V_{S2} = 2.426 V$

d)



$g_{m1} = 3g_{m2}$

e) $R_{IN} = R_B \,//\, (\beta+1)(r_{e1} + R_E)$

f) $-\dfrac{V_{OUT}}{R_D} = g_{mQ1}V_{be1} + g_{m3}\, V_{gs_{M3}} = g_{mQ1} + g_{m3}V_{g3}$

$V_3 = V_S\left(\dfrac{R_E}{r_{e1} + R_E}\right)$

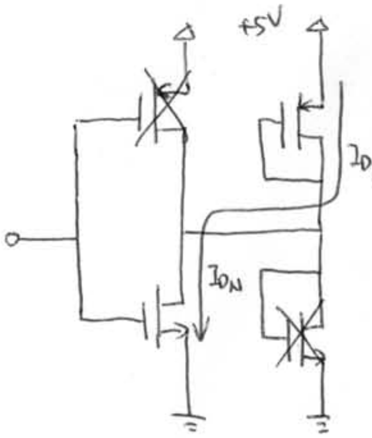
$V_{be1} = V_S\left(\dfrac{r_{e1}}{r_{e1} + R_E}\right)$

$\dfrac{V_{out}}{V_S} = -R_D\left(g_{mQ1}\dfrac{r_{e1}}{r_{e1}+R_E} + g_{mM3}\cdot\dfrac{R_E}{R_E + r_{e1}}\right)$

Q.7.

a). $\underline{V_{OL}}$

Input high, M_N : Triode M_{PL} SAT. (M_{NL}, M_P cut off)



$I_{D_N} = k_N' \frac{W}{L} \left[(V_{GS} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$

$= 8m \left[(5-1) V_{out} - \frac{1}{2} V_{out}^2 \right]$

$I_{DPL} = 0.306 \times \frac{1}{2} \times 8m \, (V_{out} - 5 + 1)^2$

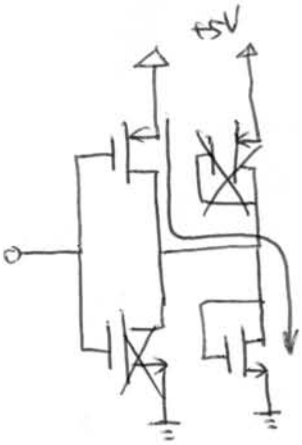
$I_{DN} = I_{DPL}$

$4V_{out} - \frac{1}{2} V_{out}^2 = \frac{1}{2} (V_{out} - 4)^2 \times 0.306.$

V_{OL}: $\boxed{V_{out} = 0.5V}$ ← Have to go back to check the status of each FET.

$\underline{V_{OH}}$

Input Low, M_P: Triode, M_{NL} SAT, (M_{PL}, M_N cut off)



$I_{Dp} = 8m \left[(-5+1)(V_{OH} - 5) - \frac{1}{2} (V_{OH} - 5)^2 \right]$

$= 8m \left[-4(V_{OH} - 5) - \frac{1}{2} (V_{OH} - 5)^2 \right]$

$I_{DNL} = \frac{1}{2} \times 8m \times 0.306 \left[(V_{OH} - 1)^2 \right]$

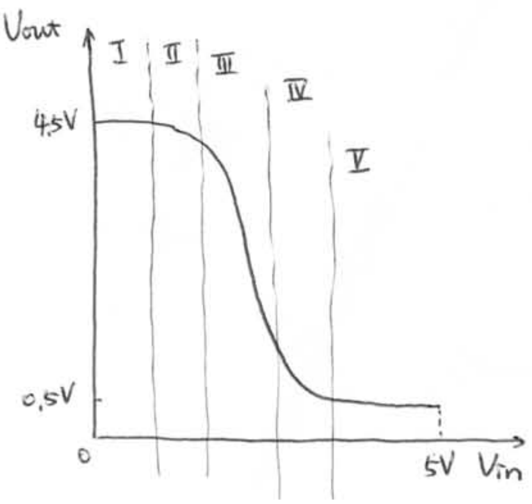
$I_{Dp} = I_{DNL}.$

$-4V_{OH} + 20 - \frac{1}{2} (V_{OH}^2 - 10V_{OH} + 25) = \frac{1}{2} \times 0.306 (V_{OH}^2 - 2V_{OH} + 1)$

$-4V_{OH} + 20 - 0.5V_{OH}^2 + 5V_{OH} - 12.5 = 0.153 (V_{OH}^2 - 2V_{OH} + 1)$

$\boxed{V_{OH} = 4.5V}$

b)



	I	II	III	IV	V
M_P	T	T	S	S	C
M_N	C	S	S	T	T
M_{PL}	C	C/S	S	S	S
M_{NL}	S	S	S	C/S	C

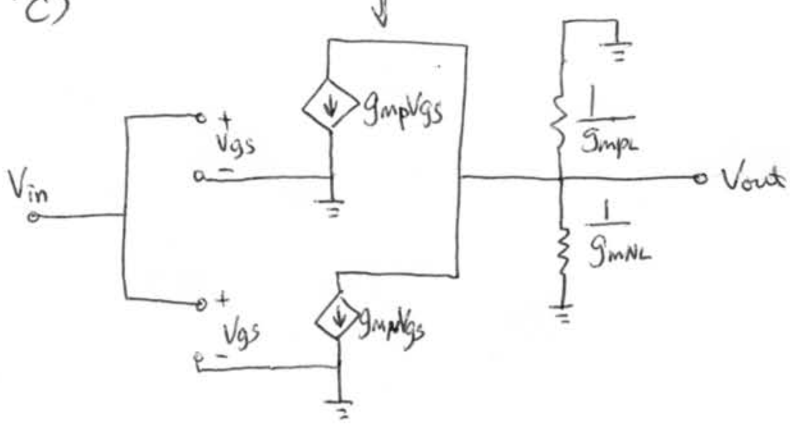
T: triode

S: Saturation

C: cutoff.

c)

T- model here is ok



d)

$$V_{out} = -2\, g_{mN} V_{gs} \times \left(\frac{1}{g_{mPL}} \,\|\, \frac{1}{g_{mNL}}\right)$$

$$\frac{V_{out}}{V_{in}} = -2\, g_{mN} \times \frac{1}{2 g_{mPL}} = -\frac{g_{mN}}{g_{mPL}}$$

$$V_{out} = 2.5V + \left(-10 \times \frac{g_{mN}}{g_{mPL}} \sin\omega t\right) mV \quad \Leftarrow$$

DC: $V_{IN} = 2.5V$. $g_{mN} = g_{mp} = g_{mPL} = g_{mNL}$.

$$V_{out} = 2.5V + (-10 \sin\omega t)$$

$$= 2.5V - 10 \sin\omega t$$