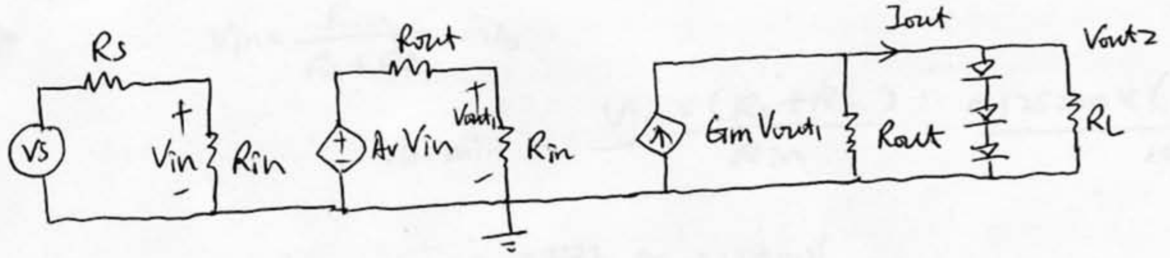
Q 1.

a)



b) All diodes OFF.

$$V_{in} = \frac{R_{in}}{R_{in}+R_s} \cdot V_s \quad ①$$

$$V_{out_1} = \frac{A_v R_{in}}{R_{out}+R_{in}} \cdot V_{in} \quad ②$$

$$I_{out} = \frac{R_{out}}{R_{out}+R_L} \times G_m V_{out_1} \quad ③.$$

Combine ①②③

$$I_{out} = \frac{R_{out}}{R_{out}+R_L} \times G_m \cdot \frac{A_v \cdot R_{in}}{R_{out}+R_{in}} \times \frac{R_{in}}{R_{in}+R_s} V_s$$

$$G_{m\text{-}tot} = \frac{I_{out}}{V_s} = \frac{R_{in}^2 R_{out} \cdot A_v G_m}{(R_{out}+R_L)(R_{in}+R_{out})(R_{in}+R_s)}$$

Since $R_{in} = R_{out}$

$$G_{m\_tot} = \frac{R_{in}^2 A_v G_m}{2(R_{in}+R_s)(R_{out}+R_L)}$$

c) $I_d = 5mA \quad V_{out2} = 3 \times 0.7 = 2.1V.$

$$I_L = \frac{V_{out2}}{R_L} = \frac{2.1V}{2K} = 1.05mA.$$

$$I_{out} = I_d + I_L = 5 + 1.05 = 6.05mA.$$

(Note that since diodes are ON, the equation derived in part b) is not valid any more.)

The current flow through the output Resistance Rout2 is I_{Rout}

$$I_{Rout} = \frac{V_{out2}}{R_{out}} = \frac{2.1}{10K} = 0.21mA.$$

$$\therefore G_m V_{out_1} = I_{Rout} + I_{out} = 6.26mA.$$

$$\Rightarrow V_{out1} = \frac{6.26m}{500m} = 0.01252V = 12.52mV.$$

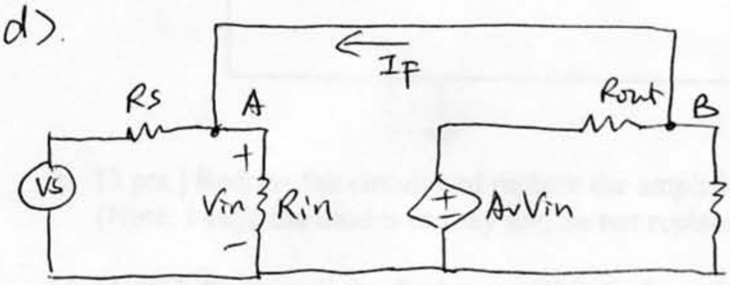
$$A_v V_{in} = 2V_{out1} \qquad V_{in} = \frac{2V_{out1}}{A_v} = 0.1252mV.$$

$$V_{in} = \frac{R_{in}}{R_s + R_{in}} \cdot V_s.$$

$$\therefore V_{s-min} = \frac{V_{in} \times (R_s + R_{in})}{R_{in}} = \frac{0.1252m \times (10K + 1K)}{10K}$$

$$V_{s-min} = 0.13772 \approx 0.138mV.$$

d).



KCL @ node A. KCL @ nod B.

$$\frac{V_s - V_{in}}{R_s} + I_F = \frac{V_{in}}{R_{in}} \qquad \frac{A_v V_{in} - V_{out}}{R_{out}} = \frac{V_{out}}{R_L} + I_F$$

$$\Downarrow$$

$$I_F = \frac{V_{in}}{R_{in}} - \frac{V_s - V_{in}}{R_s} \qquad I_F = \frac{A_v V_{in} - V_{out}}{R_{out}} - \frac{V_{out}}{R_L}$$

$$\frac{V_{in}}{R_{in}} - \frac{V_s - V_{in}}{R_s} = \frac{A_v V_{in} - V_{out}}{R_{out}} - \frac{V_{out}}{R_L}$$

$$V_{in} = V_{out}$$

$$\frac{1}{R_{in}} - \frac{\frac{V_s}{V_{out}} - 1}{R_s} = \frac{A_v - 1}{R_{out}} - \frac{1}{R_L}.$$

$$\frac{1}{R_{in}} - \frac{A_v - 1}{R_{out}} + \frac{1}{R_L} + \frac{1}{R_s} = \frac{1}{R_s} \cdot \frac{V_s}{V_{out}}.$$

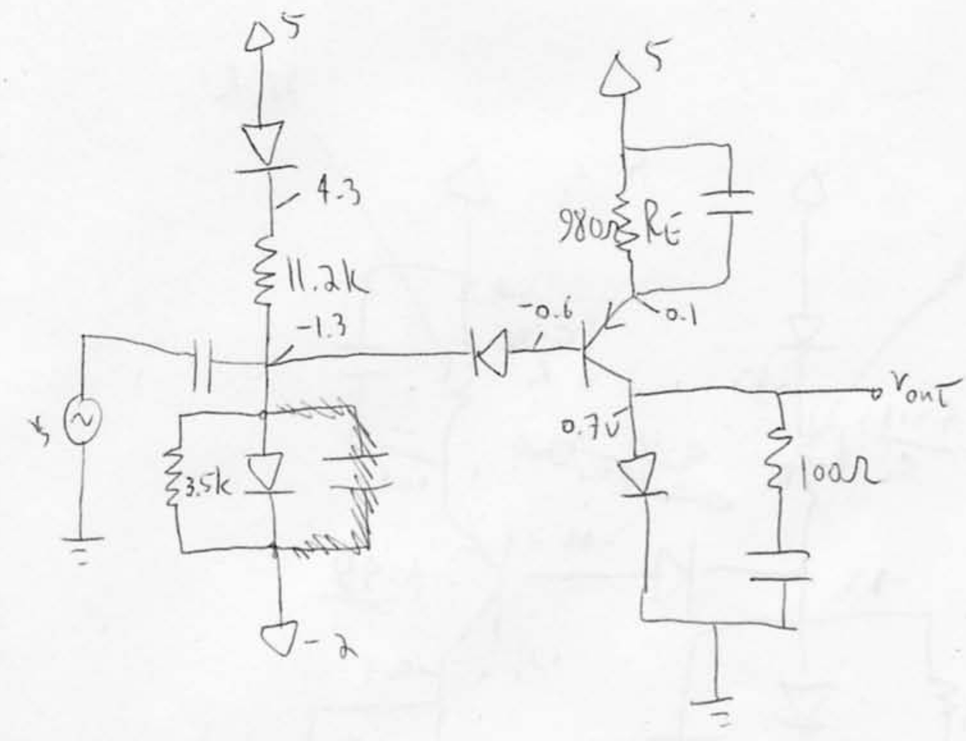
$$\frac{V_{out}}{V_s} = \frac{1}{R_s}\left[\frac{1}{R_{in}} + \frac{1}{R_L} + \frac{1}{R_s} - \frac{A_v - 1}{R_{out}}\right]^{-1}$$

Put all numbers in:

$$\frac{V_{out}}{V_s} = \frac{1}{1K}\left[\frac{1}{10K} + \frac{1}{2K} + \frac{1}{1K} - \frac{200-1}{10K}\right]^{-1}$$

Q2:

a)



-assume all ON

$$I_E = \frac{5-0.1}{980} = 5mA \rightarrow I_{D4} = \alpha I_E = \boxed{4.9mA}$$

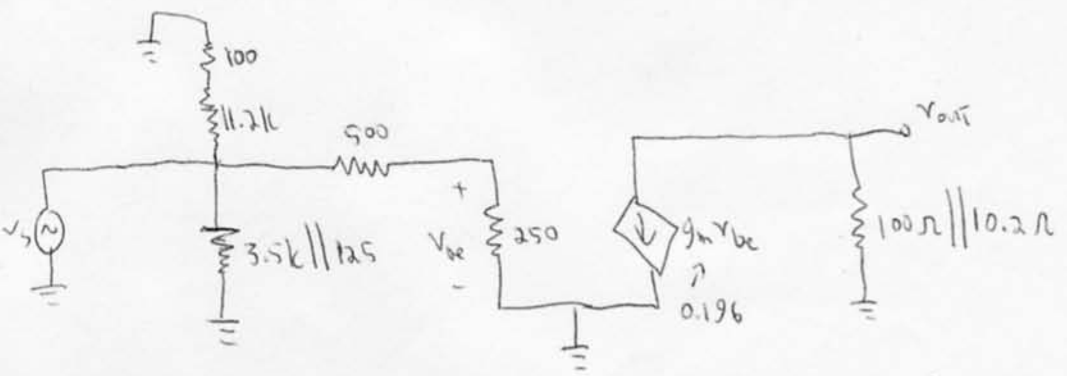
$$I_{D3} = \frac{I_E}{\beta+1} = \boxed{0.1mA}$$

$$I_{D1} = I_{11k} = \frac{4.3-(-1.3)}{11.2k} = \boxed{0.5mA} \quad \text{and} \quad I_{3.5k} = \frac{0.7}{3.5k} = 0.2mA$$

$$\therefore I_{D2} = I_{D3} + I_{D1} - I_{3.5k} = \boxed{0.4mA}$$

b) require $\frac{0.7}{R_2} \le 0.6mA$ so $R_2 \ge 1.167k$

c)



d) $\frac{v_{out}}{v_s} = \left(\frac{250}{750}\right)\left(-0.196 \times 9.256\right) = -0.605 \ V/V$

e) decrease → load in parallel with resistances!

Question 3

a) Assume saturation for all transistors

① $I_{M1} = I_{M2} = \frac{1}{2} K_p' \frac{W}{L} (V_{GS} - V_t)^2$

$$1mA = \frac{1}{2} \cdot 250 \frac{\mu A}{V^2} \cdot 12.5 (V_{GS} + 0.7)^2$$

$$\Rightarrow V_{GS} = -1.5V$$

$$V_1 = V_{CM} - V_{GS} = 3 + 1.5 = \boxed{4.5V}$$

② $I_{M3} = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_t)^2$

$$1mA = \frac{1}{2} \cdot 500 \frac{\mu A}{V^2} \cdot 4 (V_{GS} - 0.7)^2$$

$$\Rightarrow V_{GS} = 1.7V$$

$$V_2 = \boxed{1.7V}$$

③ $V_{DS1} = V_{DS2} = -2.8V < V_{GS1} - V_t < -1.5V - 0.7V < -2.2V$ OK!

$V_{DS3} = V_{DS4} = V_{GS3} = 1.7V > V_{GS3} - V_t > 1.7V - 0.7V > 1V$ OK!

All transistors in saturation!

b) For M2

Boundary at $V_{DS2} = V_{GS2} - V_{tp}$

$$V_3 - V_1 = -0.8V$$

$$\Rightarrow V_3 = 3.7V$$

For M4

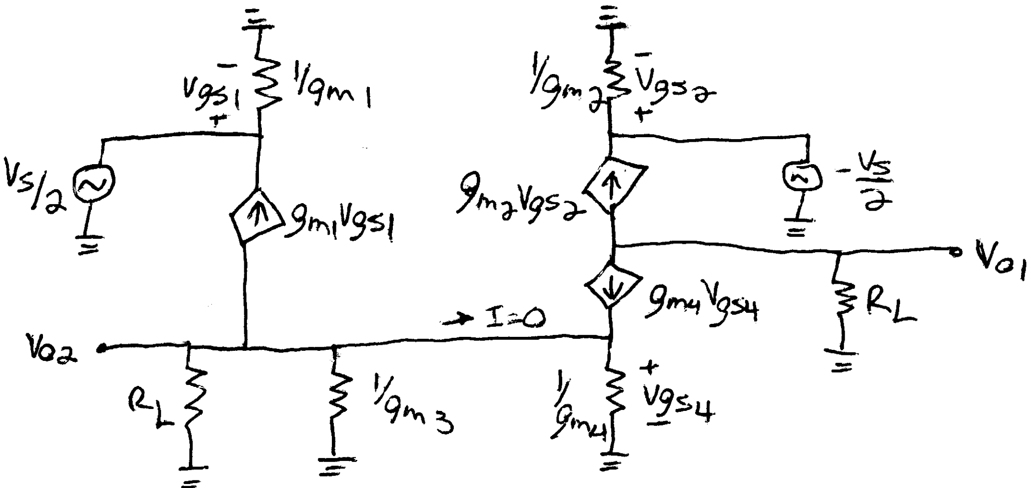
Boundary at $V_{DS4} = V_{GS4} - V_{tn}$

$$V_3 - 0 = 1V$$

$$\Rightarrow V_3 = 1V$$

$\therefore$ $1V \leq V_3 \leq 3.7V$ for saturation.

c)



d) $g_{m1} = \sqrt{2K'_p}\sqrt{\frac{W}{L}}\sqrt{I_{M1}}$ $\qquad g_{m2} = g_{m1}$

$= \sqrt{2 \cdot 250\frac{\mu A}{V^2}}\sqrt{12.5}\sqrt{1mA}$

$= 2.5\times10^{-3}\ \frac{A}{V}$

$g_{m3} = \sqrt{2K'_n}\sqrt{\frac{W}{L}}\sqrt{I_{M3}}$ $\qquad g_{m4} = g_{m3}$

$= \sqrt{2 \cdot 500\frac{\mu A}{V^2}}\sqrt{4}\sqrt{1mA}$

$= 2\times10^{-3}\ \frac{A}{V}$

e) $V_{gs1} = \frac{V_s}{2}$

$V_{o2} = -g_{m1}V_{gs1}\left(R_L \,//\, 1/g_{m3}\right)$

$\frac{V_{o2}}{V_s} = -\frac{g_{m1}}{2}\left(\frac{R_L}{1+g_{m3}R_L}\right) = \frac{-g_{m1}R_L}{2(1+g_{m3}R_L)} = -\frac{6.25}{11} = -0.57\frac{V}{V}$

f) $V_{gs2} = -\frac{V_s}{2}$ $\qquad V_{gs4} = V_{o2}$

$V_{o1} = -(g_{m2}V_{gs2} + g_{m4}V_{gs4})\cdot R_L$

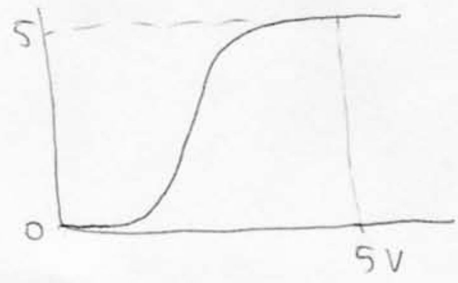
$= \left(\frac{g_{m2}}{2} + \frac{g_{m1}g_{m4}R_L}{2(1+g_{m3}R_L)}\right)\cdot R_L = (1.25\times10^{-3} + 1.136\times10^{-3})\cdot 5000 = 11.93\frac{V}{V}$

Q4

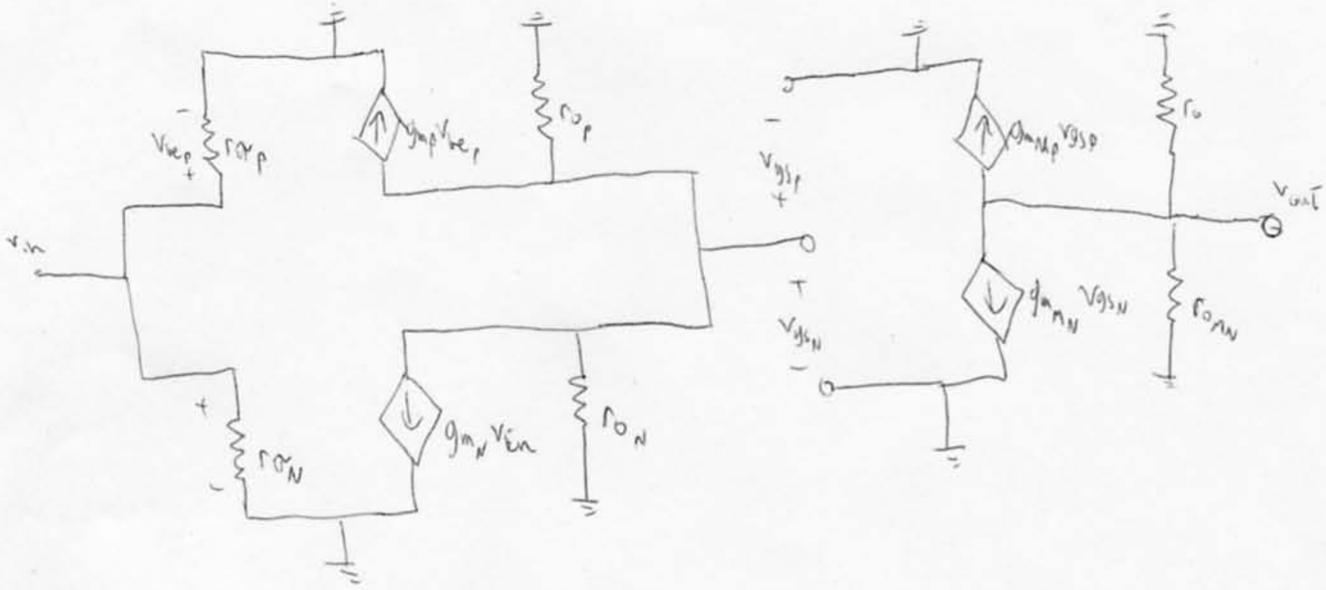
a) Q_P - sat $\quad M_P$ - cutoff

Q_N - cutoff $\quad M_N$ - Triode

b)



c)



d) $v_{gsN} = v_{gsP} = -g_{mN} r_{oN} v_{in}$ and $v_{out} = -g_{m_{M_N}} v_{gs_N} r_{oM_N}$

$$\frac{v_{out}}{v_{in}} = g_{m_N} \cdot g_{m_{M_N}} \cdot r_{oN} \cdot r_{oM_N}$$

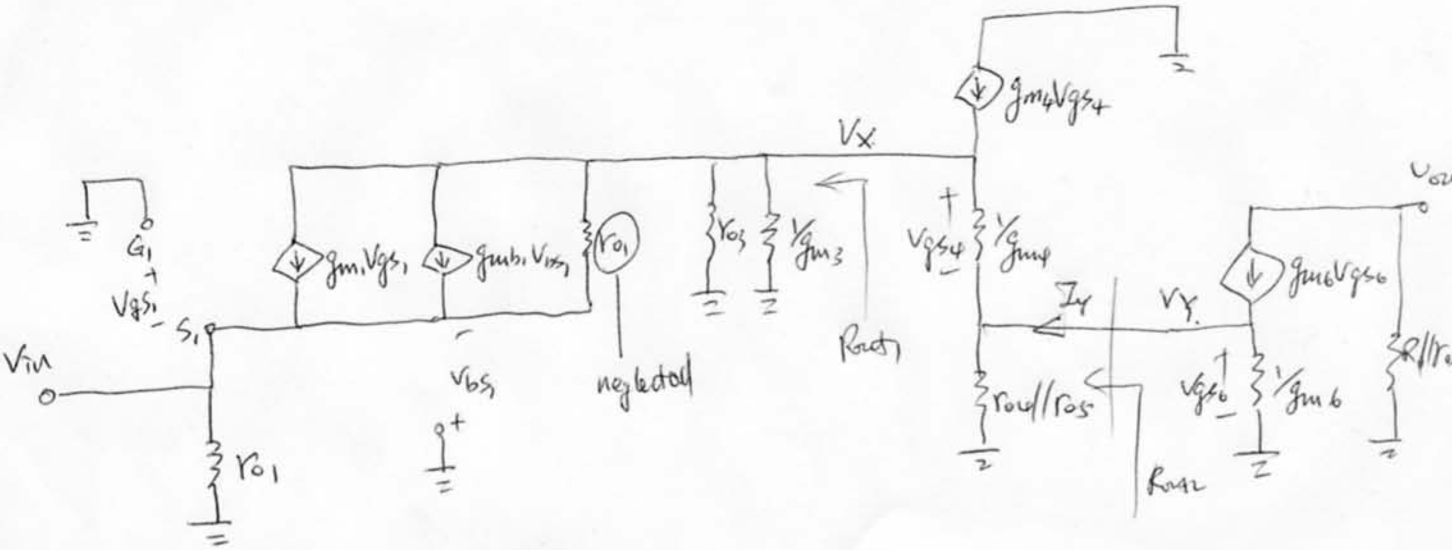
e) $g_{m_N} \cdot r_{o_N} = \frac{V_A}{V_T}$; independent of biasing!

$g_{mM} \cdot r_{oM} = \frac{1}{\lambda(V_{GS}-V_t)}$ depends on the DC biasing...

#5

a) CGA CDA CSA.

b)



c) $\frac{V_x}{V_{in}} = \frac{-(g_{m1} V_{gs1} + g_{mb1} V_{bs1})(r_{o3} // \frac{1}{g_{m3}})}{V_{in}}$

& $V_{gs1} = -V_{in}$ $V_{bs1} = -V_{in}$ $\therefore \frac{V_x}{V_{in}} = +(g_{m1} + g_{mb1})(r_{o3} // \frac{1}{g_{m3}})$

d). $\frac{V_Y}{V_X} = \frac{g_{m4} V_{gs4} \times (r_{o4} // r_{o5})}{V_X} = \frac{g_{m4}(V_X - V_Y)(r_{o4} // r_{o5})}{V_X}$

$\Rightarrow \frac{V_Y}{V_X} = \frac{g_{m4} r_{o4} // r_{o5}}{1 + g_{m4} r_{o4} // r_{o5}}$

e) $\frac{V_{out}}{V_Y} = g_{m6} R_1 // r_{o6}$

f) $R_{out1} = r_{o3} // \frac{1}{g_{m3}}$

g) $R_{out2} = \frac{V_Y}{I_Y} = \frac{(I_Y + g_{m4} V_{gs4}) \cdot r_{o4} // r_{o5}}{I_Y} = \frac{(I_Y - g_{m4} V_Y)(r_{o4} // r_{o5})}{I_Y}$

$= (1 - g_{m4} \frac{V_Y}{I_Y})(r_{o4} // r_{o5})$

$R_{in} = \frac{V_Y}{I_Y} = \frac{(r_{o4} // r_{o5})}{1 + g_{m4} r_{o4} // r_{o5}}$

Question 6

a) Edge of Triode/saturation

$$V_{DS} = V_{GS} - V_t = -1.3 + 0.6 = -0.7$$

$$\therefore V_{D_1} = 2.6V$$

$$I_D = \frac{1}{2} K_p' \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$1mA = \frac{1}{2} \cdot 0.47285 \cdot K_p' \frac{W}{L} \qquad K_p' = 500 \frac{\mu A}{V^2}$$

$$\frac{W_1}{L_1} = \boxed{8.46}$$

b) saturation

c) ① $I_{M_2} = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_t)^2$

$$1mA = \frac{1}{2} \cdot 1 \frac{mA}{V^2} \cdot 1 \, (V_{GS} - V_t)^2$$

$$= V_{GS_{M_2}} = 2.01\,V$$

$$V_{DS} = 2.6 - 0.99 = 1.61 \geq V_{GS} - V_t$$
$$\geq 2.01 - 0.6$$
$$\geq 1.41$$
ok!

② $V_{S_2} = V_{Gn} - V_{GS_{M_2}} = 3 - 2.01 = 0.99\,V$

③ $V_{EQ_2} = V_{S_2} - 0.7V = 0.29V$

④ $I_{EQ_2} = \frac{0.29V}{200\Omega} = 1.45mA = I_{CQ_1} \quad (\beta = 100)$

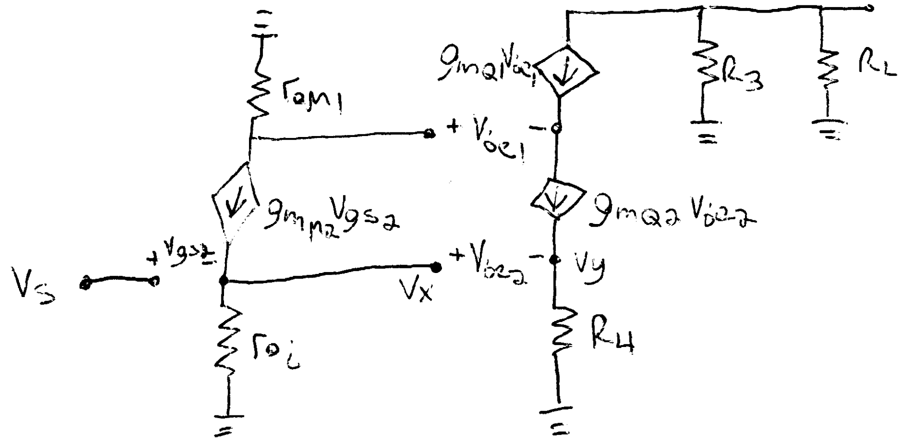
⑤ $V_{max\text{-}sym} = \frac{3.3 + 2.1}{2} = 2.7\,V$

⑥ ~~$R_3 = \frac{2.7V}{1.45mA}$~~ $\Rightarrow V_{CC} - R_3 I_{C_1} = 2.7$

$$\Rightarrow R_3 = \frac{0.6}{1.45mA} = 413.8\,\Omega$$

d)

$\beta = \infty \Rightarrow r_\pi = \infty$



e)

① $V_{OUT} = -g_{mQ1} V_{be1} (R_3 // R_L)$

② $g_{mQ1} V_{be1} = g_{mQ2} V_{be2} \Rightarrow V_{be1} = \frac{g_{mQ2}}{g_{mQ1}} V_{be2}$

③ $V_{be2} = V_x - V_y$

④ $V_y = g_{mQ2}(V_x - V_y) R_4 \Rightarrow V_y = \frac{g_{mQ2} R_4}{1 + g_{mQ2} R_4} V_x$

⑤ $V_{gs2} = V_s - V_x$

⑥ $V_x = g_{mM2}(V_s - V_x) r_{oi} \Rightarrow V_x = \frac{g_{mM2} r_{oi}}{1 + g_{mM2} r_{oi}} V_s$

⑦ $V_{be2} = \frac{g_{mM2} r_{oi}}{1 + g_{mM2} r_{oi}} \left[1 - \frac{g_{mQ2} R_4}{1 + g_{mQ2} R_4} \right] V_s$

⑧ $\frac{V_{OUT}}{V_S} = -g_{mQ1} (R_3 // R_L) \frac{g_{mQ2}}{g_{mQ1}} \left(\frac{g_{mM2} r_{oi}}{1 + g_{mM2} r_{oi}} \right) \left(1 - \frac{g_{mQ2} R_4}{1 + g_{mQ2} R_4} \right)$

$= -g_{mQ2} (R_3 // R_L) \left(\frac{g_{mM2} r_{oi}}{1 + g_{mM2} r_{oi}} \right) \left(1 - \frac{g_{mQ2} R_4}{1 + g_{mQ2} R_4} \right)$