EX.1 . Which of the following impulse responses correspond(s) to stable LTI systems?

(a) $h_1(t) = e^{-(1-2j)t} u(t)$ (b) $h_2(t) = e^{-t} \cos(2t) u(t)$

(c) $h_3[n] = n \cos(\frac{\pi}{4} n) u[n]$ (d) $h_4[n] = 3^n u[-n+10]$

EX. 2. Consider an LTI system with input x and output y which are related by: $\frac{d}{dt} y(t) + 4y(t) = x(t)$

The system satisfies the condition of initial rest.

(a) If $x(t) = e^{(-1+3j)t} u(t)$, what is $y(t)$?

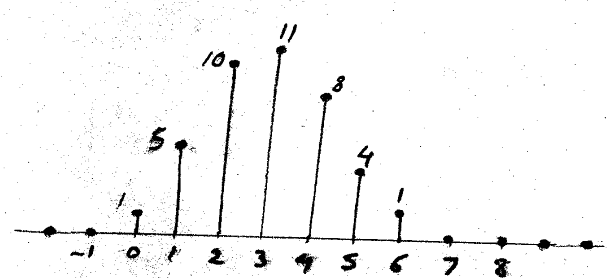
(b) Note that $Re\{x(t)\}$ will satisfy equation above, with $Re\{y(t)\}$. Determine the output y if $x(t) = e^{-t} \cos(3t) u(t)$

EX 3. Consider the cascade interconnection of 3 causal systems:

$$x[n] \longrightarrow \boxed{h_1[n]} \longrightarrow \boxed{h_2[n]} \longrightarrow \boxed{h_2[n]} \longrightarrow y[n]$$

where $h_2[n] = u[n] - u[n-2]$

and the overall impulse response is shown by:



(a) Find the impulse response $h_1[n]$.

(b) Find the response of the overall system to the input

$$x[n] = \delta[n] - \delta[n-1]$$

Ex. 1. (a) absolutely integrable?

$$\int_{-\infty}^{\infty} |h_1(\tau)|\, d\tau = \int_0^{\infty} e^{-\tau}\, d\tau = 1 \checkmark \implies \text{stable} \checkmark$$

(b) $$\int_{-\infty}^{\infty} |h_2(\tau)|\, d\tau = \int_0^{\infty} e^{-\tau} |\cos 2\tau|\, d\tau \leq \int_0^{\infty} e^{-\tau} \cdot 1\, d\tau$$

$$\leq 1 \implies \text{stable} \checkmark$$

(c) absolutely summable?

$$\sum_{k=-\infty}^{\infty} |h_3[k]| = \sum_{k=-\infty}^{\infty} |k \cos(\tfrac{\pi}{4} k)\, u[k]| = \underbrace{\sum_{k=0}^{\infty} k\, |\cos(\tfrac{\pi}{4} k)|}_{\text{increasing}}$$

$\implies$ not absolutely summable $\longrightarrow$ not stable

d) $$\sum_{k=-\infty}^{\infty} |3^k u[-k+10]| = \sum_{k=-\infty}^{10} 3^k = \frac{3^{10}}{1-\frac{1}{3}} = \frac{3^{11}}{2}$$

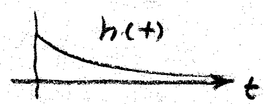
$\implies$ stable $\checkmark$

EX.2 (a) Determine the impulse response $h(t)$ first :

$$\frac{dh(t)}{dt} + 4h(t) = \delta(t) \Longrightarrow e^{4t}\frac{dh}{dt} + 4e^{4t}h(t) = e^{4t}\delta(t)$$

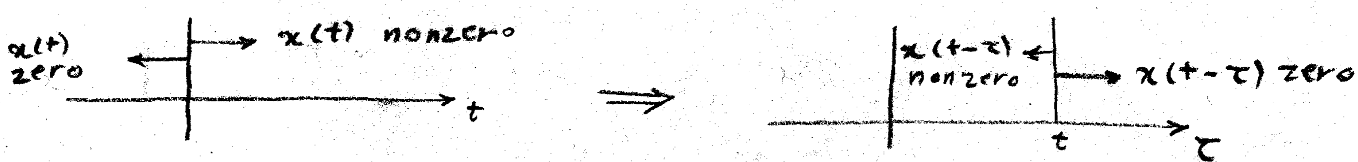
$$\Longrightarrow \frac{d(e^{4t}h)}{dt} = e^{4t}\delta(t) \Longrightarrow e^{4t}h(t) = \int_{-\infty}^{t} e^{4\tau}\delta(\tau)\,d\tau = u(t)$$

$$\Longrightarrow h(t) = e^{-4t}u(t)$$



Now : $y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{t} x(t-\tau)\,h(\tau)\,d\tau \quad \text{but} \quad x(t) = e^{(-1+3j)t}u(t)$$



$t > 0$:

$$y(t) = \int_0^t e^{(-1+3j)(t-\tau)}\,e^{-4\tau}\,d\tau$$

$$= \int_0^t e^{(-1+3j)t + (-3-3j)\tau}\,d\tau$$

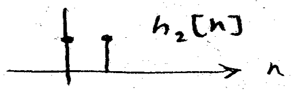
$$= e^{(-1+3j)t}\left[\frac{e^{(-3-3j)t} - 1}{-3-3j}\right] \quad \text{when } t > 0$$

$$\Longrightarrow y(t) = \frac{e^{-4t} - e^{(-1+3j)t}}{-3(1+j)}\,u(t) = \frac{1-j}{6}\left[e^{(-1+3j)t} - e^{-4t}\right]u(t)$$

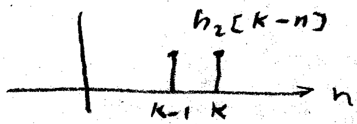
(b) The output will be the real part of y obtained in a :

$$y(t) = \frac{1}{6}\left[e^{-t}\cos 3t + e^{-t}\sin 3t - e^{-4t}\right]u(t)$$

EX. 3. (a) We are given $h_2[n] = \delta[n] + \delta[n-1]$



$$h_2 * h_2 = \sum_{k=-\infty}^{\infty} h_2[n] h_2[k-n]$$



$$= \sum_{k=0}^{2} h_2[n] h_2[k-n]$$

$$= \delta[n] + 2\delta[n-1] + \delta[n-2]$$

$$h[n] = h_1[n] * \left[h_2[n] * h_2[n] \right]$$

$$= h_1[n] + 2h_1[n-1] + h_1[n-2]$$

$$\Longrightarrow \quad h[0] = h_1[0] + 2h_1[-1] + h_1[-2]$$

h_1 : causal

$$= h_1[0] + 0 + 0 \Longrightarrow h_1[0] = 1$$

$$h[1] = h_1[1] + 2h_1[0] \Longrightarrow h_1[1] = 3$$

$$h[2] = h_1[2] + 2h_1[1] + h_1[0] \Longrightarrow h_1[2] = 3$$

$$h[3] = h_1[3] + 2h_1[2] + h_1[1] \Longrightarrow h_1[3] = 2$$

$$h[4] = h_1[4] + 2h_1[3] + h_1[2] \Longrightarrow h_1[4] = 1$$

$$h[5] = h_1[5] + 2h_1[4] + h_1[3] \Longrightarrow h_1[5] = 0$$

$h_1[n] = 0$ for $n < 0$, $n \geq 5$

(b) $y[n] = h[n] - h[n-1]$