

6.003: Signals and Systems — Spring 2004

TUTORIAL 6 SOLUTIONS

Tuesday, March 16, 2004

Problem 6.1

(a)

$$X_a(e^{j\omega}) = \begin{cases} \frac{1}{2j} \left(\frac{1}{1 - \frac{1}{2}e^{j(\omega - \frac{\pi}{4})}} - \frac{1}{1 - \frac{1}{2}e^{j(\omega + \frac{\pi}{4})}} \right), & |\omega| \leq \pi \\ X_a(e^{j(\omega - 2\pi k)}), & \text{for all integers } k \end{cases}$$

(b)

$$X_b(e^{j\omega}) = \begin{cases} \frac{1}{2}, & \frac{3\pi}{10} < |\omega| < \frac{7\pi}{10} \\ 0, & \frac{7\pi}{10} < |\omega| \leq \pi \\ X_b(e^{j(\omega - 2\pi k)}), & \text{for all integers } k \end{cases}$$

Problem 6.2

(a)

$$x_a[n] = \frac{1}{2}\delta[n] + \frac{1}{4}\delta[n - 2] + \frac{1}{4}\delta[n + 2]$$

(b)

$$x_b[n] = (-2)^n u[-n] * \left(-\frac{1}{2}\right)^n u[n] = \frac{4}{3} \left(-\frac{1}{2}\right)^{|n|}$$

Problem 6.3

(a)

$$x_a[n] = -2 \left(\frac{1}{4}\right)^n u[n] + 4 \left(\frac{1}{2}\right)^n u[n]$$

(b)

$$x_b[n] = -2(n+1) \left(\frac{1}{2}\right)^n u[n] - 6 \left(\frac{1}{2}\right)^n u[n] + 9 \left(\frac{3}{4}\right)^n u[n]$$

(c)

$$x_c(t) = \frac{1}{2}e^{-3t}u(t) + \frac{1}{2}e^{-t}u(t)$$

(d)

$$x_d(t) = \delta(t) + 16e^{-3t}u(t) - 23e^{-4t}u(t)$$

Problem 6.4

(a)

$$H(j\omega) = \frac{j\omega + 2}{(j\omega + 3)(j\omega + 1)}$$

(b)

$$h(t) = \frac{1}{2}e^{-3t}u(t) + \frac{1}{2}e^{-t}u(t)$$

(c)

$$y(t) = -\frac{1}{2}e^{-3t}u(t) + \frac{1}{2}e^{-t}u(t)$$

Problem 6.5

$$x(t) = e^{-4t}u(t)$$

Problem 6.6

(a)

$$y_1(t) = \frac{1}{2} \sin(3t - 3)$$

(b)

$$y_2(t) = \left(\frac{\sin 2(t-1)}{\pi(t-1)} \right)^2$$