MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems — Spring 2004

TUTORIAL 4 SOLUTIONS

Tuesday, March 2, 2004

Problem 4.1

(a) $H(j\omega) = \frac{2}{5j\omega - (2\omega^2 + 3)}$

(b) (i) $y_a(t) = A\cos(3\pi t + B)$
$A = \frac{2}{\sqrt{(15\pi)^2 + (18\pi^2 + 3)^2}}$
$B = \pi + \tan^{-1}\left(\frac{5\pi}{6\pi^2 + 1}\right)$

(ii) $y_b(t) = A\sin(\frac{3\pi}{4}t + B) + C\cos(\pi t + D)$
$A = \frac{2}{\sqrt{(\frac{15\pi}{4})^2 + (\frac{9\pi^2}{8} + 3)^2}}$
$B = \tan^{-1}\left(\frac{30\pi}{9\pi^2 + 24}\right)$
$C = \frac{2}{\sqrt{(5\pi)^2 + (2\pi^2 + 3)^2}}$
$D = \frac{\pi}{3} + \tan^{-1}\left(\frac{5\pi}{2\pi^2 + 3}\right)$

(c) $b_k = H(j\frac{2\pi}{T}k)a_k = H(j\omega_0 k)a_k$

Problem 4.2

(a) $H(e^{j\omega}) = \frac{2}{2 + 5e^{-j\omega} - 3e^{-2j\omega}}$

(b) (i) $y_a[n] = A\cos(\frac{\pi}{3}n + B)$
$A = \frac{2}{\sqrt{39}}$
$B = \tan^{-1}\left(\frac{\sqrt{3}}{6}\right)$

(ii) $y_b[n] = A\sin(\frac{3\pi}{4}n + B) + C\cos(\frac{\pi}{4}n + D)$
$A = \frac{4}{\sqrt{(4 - \sqrt{2})^2 + (6 + 5\sqrt{2})^2}}$
$B = \tan^{-1}\left(\frac{6 + 5\sqrt{2}}{4 - \sqrt{2}}\right)$
$C = \frac{4}{\sqrt{(4 + \sqrt{2})^2 - (6 - 5\sqrt{2})^2}}$
$D = \frac{\pi}{3} - \tan^{-1}\left(\frac{6 - 5\sqrt{2}}{4 + \sqrt{2}}\right)$

(c) $b_k = H(e^{j\frac{2\pi}{T}k})a_k = H(e^{j\omega_0 k})a_k$

Problem 4.3 $y(t) = \frac{8}{3} + 2\sqrt{3}\cos(\frac{\pi}{3}t)$

Problem 4.4 $b_k = \begin{cases} 0, & k = -1 + 10m, 10m, 1 + 10m, \text{ where m is an integer} \\ a_k, & otherwise \end{cases}$

Problem 4.5

(a) $\frac{L}{R}\frac{d^2}{dt^2}y(t) + \frac{d}{dt}y(t) + \frac{1}{RC}y(t) = \frac{d}{dt}x(t)$

(b) $H(j\omega) = \frac{RCj\omega}{1 - LC\omega^2 + RCj\omega}$

(c) $H(j\omega) = \frac{RCj\omega}{1 - LC\omega^2 + RCj\omega}$ (the solution is verified)

(d) This is a bandpass filter

(e) $y(t) = A(2V)\cos(3\pi t + B)$
     $A = \frac{3RC\pi}{\sqrt{(1 - 9LC\pi^2)^2 + (3RC\pi)^2}}$
     $B = \frac{5\pi}{6} - \tan^{-1}\left(\frac{1 - 9LC\pi^2}{3RC\pi}\right)$

Problem 4.6 $y[n] = A\sin(\frac{2\pi}{3}n + B) + \left[\frac{1 - \frac{1}{2}^{n-1}}{1 - \frac{1}{2}}\right]u[n - 2]$
$A = \frac{2}{\sqrt{7}}$
$B = -\tan^1\left(\frac{\sqrt{3}}{5}\right)$

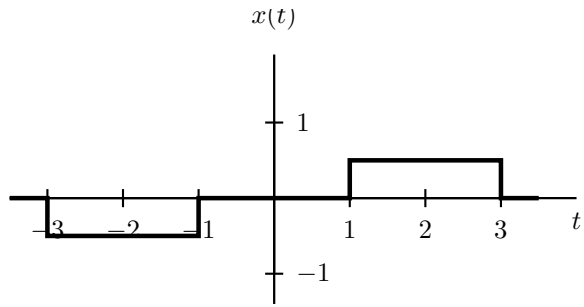**Problem 4.7**   The transform is $X(j\omega) = \frac{2\pi \sin \omega}{\omega}$

Problem 4.8

(a) $\theta(j\omega) = \omega + \frac{\pi}{2}$

(b) $X(\omega 0) = 0$

(c) $\int_{-\infty}^{+\infty} X(j\omega)d\omega = 2\pi$

(d) $\int_{-\infty}^{+\infty} |X(j\omega)|^2 \, d\omega = 8\pi$

(e)