MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems — Spring 2004

Tutorial 1 Solutions

Monday and Tuesday, February 9 and 10, 2004

Problem 1.1

(a) Rectangular: $\frac{1+j}{\sqrt{3}+j} = \frac{1+j}{\sqrt{3}+j} \cdot \frac{\sqrt{3}-j}{\sqrt{3}-j} = \frac{\sqrt{3}+1+j(\sqrt{3}-1)}{3+1} = \frac{(\sqrt{3}+1)+j(\sqrt{3}-1)}{4}$.
Polar: $\frac{1+j}{\sqrt{3}+j} = \frac{\sqrt{2}e^{j\pi/4}}{2e^{j\pi/6}} = \frac{1}{\sqrt{2}}e^{j\pi/12}$.
You can verify that these are equivalent.

(b) $e^j + e^{3j} = e^{2j}\left(e^{-j} + e^{j}\right) = e^{2j}(2\cos 1)$. So, the magnitude is $2\cos 1$ and the phase is 2.

(c) $(\sqrt{3}-j)^8 = (2e^{-j\pi/6})^8 = 2^8 e^{8(-j\pi/6)} = 256e^{-j4\pi/3} = -128 + j128\sqrt{3}$.

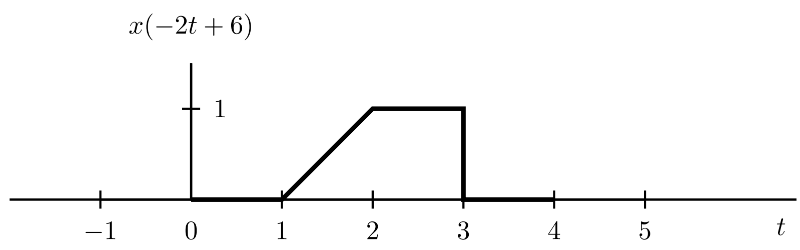
(d) $\int_0^\infty e^{-2t}\cos(\pi t)\,dt = \int_0^\infty e^{-2t}2\left(e^{j\pi t} + e^{-j\pi t}\right)\,dt = 2\left(\int_0^\infty e^{(-2+j\pi)t}\,dt + \int_0^\infty e^{(-2-j\pi)t}\,dt\right)$
$= 2\left(\frac{1}{-2+j\pi}e^{(-2+j\pi)t}|_0^\infty + \frac{1}{-2-j\pi}e^{(-2-j\pi)t}|_0^\infty\right) = \frac{2}{4+\pi^2}$.

(e) Let r be the magnitude of z and θ be the phase of z. Then, $\frac{1-z^n}{1-z} = \sum_{k=0}^{n-1} z^k = \sum_{k=0}^{n-1}\left(re^{j\theta}\right)^k = \sum_{k=0}^{n-1} r^k e^{jk\theta}$. So, $Re\left\{\frac{1-z^n}{1-z}\right\} = Re\left\{\sum_{k=0}^{n-1} r^k e^{jk\theta}\right\} = \sum_{k=0}^{n-1} r^k Re\left\{e^{jk\theta}\right\} = \sum_{k=0}^{n-1} r^k\cos(k\theta)$.

(f) There are often several ways of manipulating complex numbers, even for the simplest cases. Knowing which method solves a particular problem is easiest or most insightful is a skill that would be useful to acquire in the course of learning 6.003.

Example 1.2

All three methods produce the same result:



Problem 1.3

(a) Periodic. The period is $\frac{2\pi}{3\pi/2} = \frac{4}{3}$.

(b) Periodic. We have $2\pi(3) = \frac{3\pi}{2}(4)$, and 3 and 4 share no common factors, so the period is 4.

(c) Periodic. The period is LCM(4/3, 6) = 12.

(d) Periodic. The period is LCM(4, 6) = 12.

(e) Not periodic. 4/3 and 2π are incommensurate numbers, *i.e.* their ratio is irrational.

(f) Not periodic. There exist no integers m and N such that $3N = 2\pi m$.

(g) Periodic. Even though each signal has fundamental period 2, the fundamental period of $x_g(t)$ is 1.

Problem 1.4

(a) $x[n] = -\delta[n+1] + 2\delta[n-1] - \delta[n-2]$.

(b) $x[n] = -u[n+1] + u[n] + 2u[n-1] - 3u[n-2] + u[n-3]$.

Problem 1.5

System	Stable	Causal	Linear	Time-Invariant
a	No	Yes	Yes	Yes
b	No	No	Yes	No
c	Yes	No	Yes	No
d	Yes	Yes	No	Yes
e	Yes	Yes	No	Yes

Problem 1.6

(a) True.

(b) True.

(c) True.

(d) False. Let system 1 do $x(t) \to y(t)$ and system 2 do $y(t) \to z(t)$. If $y(t) = \mathrm{e}^{j\pi t}x(t)$ and $z(t) = \mathrm{e}^{-j\pi t}y(t)$, we see that each system is time-varying, but the overall system is $z(t) = x(t)$, which is time-invariant.