ECSE306 Midterm 1, September 30, 2008

Name:____________________________________ Student number: __________________________

1. (10 points) Please fill in "Yes" or "No" according to the systems' properties

Systems	2y[n]=x[n-1]+1	2dy(t)/dt+y(t)=2x(t)cos(2t)
Memory-less?	No	Yes
BIBO Stable?	Yes	Yes
Linear?	No	Yes
Causal?	Yes	Yes
Time-invariant?	Yes	No

2. (10 points) Please fill in "Yes" or "No" according to the signals' properties

Signals	x[n]=cos[5n]sin[6n]	$x(t)=\sin(t)e^{j3\pi t}$
Periodic?	No	No
Even?	No	No
Odd?	Yes	No
Finite-power ?	Yes	Yes
Finite-energy ?	No	No

3. (10 points) Compute y(t)=x(t)*h(t), where $x(t)= e^{-t}u(t+1)$, and $h(t)=e^{-t/2}u(t)$.

$$y(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau=\int_{-1}^{t}e^{-\tau}u(\tau+1)e^{-(t-\tau)/2}u(t-\tau)d\tau=e^{-t/2}\int_{-1}^{t}e^{-\tau/2}d\tau$$

$$=-2e^{-t/2}e^{-\tau/2}\Big|_{-1}^{t}=-2e^{-t/2}(e^{-t/2}-e^{1/2}),\qquad t\geq -1$$

4. (15 points) Compute y[n]=x[n]*h[n], where x[n]=u[n+1]+u[n+3], and h[n]= 2^{-n}u[n]. (hint: to simplify the calculation, you may apply the principle of superposition).

Answer:

Let the input be $x_1[n]$=u[n+1], then the corresponding output is:

$$y_1[n] = \sum_{k=-\infty}^{\infty} x_1[k]h[n-k] = \sum_{k=-1}^{n} u[k+1]2^{-(n-k)}u[n-k] = \sum_{k=-1}^{n} 2^{-(n-k)} = 2^{-n}\sum_{k=-1}^{n} 2^k = 2^{-n}(2^{-1} + \sum_{k=0}^{n} 2^k)$$

$$= 2^{-n}(2^{-1} + \frac{1-2^{n+1}}{1-2}) = 2^{-n}(0.5 - 1 + 2^{n+1}) = -2^{-n-1} + 2$$

$$y_1[n] = (-2^{-n-1} + 2)u[n+1]$$

For the input x[n] =u[n+1]+u[n+3], the output is

$$y[n] = y_1[n] + y_1[n+2] = (-2^{-n-1} + 2)u[n+1] + (-2^{-n-3} + 2)u[n+3]$$

5. (15 points)

5a. (10 point) Derive the impulse response of the system below, where *a* and *b* are constant, and the systems are initially at rest.

$$a\frac{dy(t)}{dt} + by(t) = \frac{dx(t)}{dt}$$

5b. (5 points) Indicate the condition for the system to be stable.

Answer:

5a. First derive the impulse response of for homogeneous equation:

$$a\frac{dy(t)}{dt} + by(t) = \delta(t)$$

The homogeneous solution is $h_a(t)=Ae^{-bt/a}u(t)$.

Substitute y(t) in the above Eq. with $h_a(t)$:

$$a[-\frac{b}{a}Ae^{-bt/a}u(t) + Ae^{-bt/a}\delta(t)] + bAe^{-bt/a}u(t) = \delta(t)$$

For the two sides to be equal, Aa=1, i.e., A=1/a. Thus, the impulse response for the homogeneous equation is $h_a(t)=(1/a)\, e^{-bt/a}u(t)$

As the input is the derivative of x(t), then according to the principle of superposition, the impulse response for the original equation is $h(t)=dh_a(t)/dt$:

$$h(t) = \frac{dh_a(t)}{dt} = \frac{d}{dt}\{\frac{1}{a}e^{-bt/a}u(t)\} = -\frac{b}{a^2}e^{-bt/a}u(t) + \frac{1}{a}e^{-bt/a}\delta(t) = -\frac{b}{a^2}e^{-bt/a}u(t) + \frac{1}{a}\delta(t)$$

5b. for the system to be stable, the zero of the characteristic polynomial should have negative real part, i.e., Re{-b/a}<0.

6. (20 points)
a. (15 point) Derive the impulse response of the system $ay[n] + by[n-1] = x[n-2]$, where a and b are constant, and the systems are initially at rest.

b. (5 points) Indicate the condition for the above system to be stable.

Answer:
a. Solve for the impulse response for the homogeneous Eq.: $ay[n] + by[n-1] = \delta[n]$

The homogeneous solution is $h_a[n] = Az^n u[n]$. Substituting it into the above Eq.:

$$aAz^n u[n] + b\,Az^{n-1} u[n-1] = \delta[n]$$

for n=0,

$$aA = \delta[0] = 1.$$

for n>=1,

$$aAz^n + b\,Az^{n-1} = 0.$$

Then A=1/a, z= - b/a, and $h_a[n] = (1/a)\,(-b/a)^n u[n]$.
According to the superposition, the impulse response for the original Eq. is

$$h[n] = h_a[n-2] = (1/a)(-b/a)^{n-2} u[n-2].$$

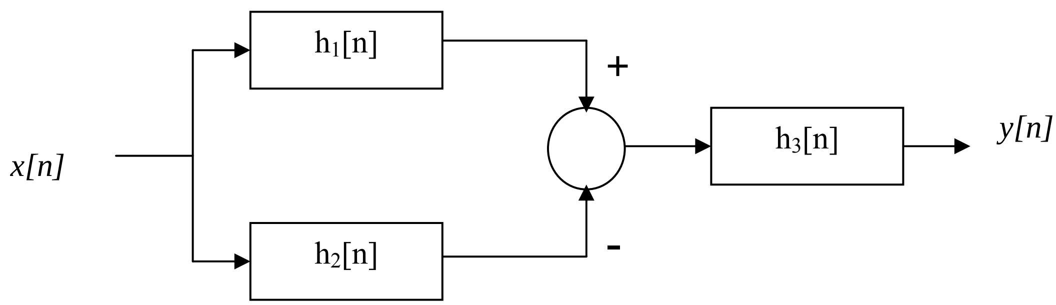
b. For the system to be stable, |-b/a|<1 should be satisfied.

7. (15 points) A system is composed of an inter connection of other 3 systems, as shown below. The impulse responses of the 3 systems are $h_1[n] = -u[n-1]$, $h_2 = \delta[n-5]$, and $h_3[n] = u[n] - u[n-1]$, respectively. Derive the impulse response of the system.

Answer:

The impulse response is $h[n] = (h_1[n] - h_2[n]) * h_3[n] = (-u[n-1] - \delta[n-5]) * (u[n] - u[n-1])$

$$h[n] = (-u[n-1] - \delta[n-5]) * \delta[n] = -u[n-1] - \delta[n-5].$$


x[n]
h1[n]
h2[n]
+
-
h3[n]
y[n]