1 a) Let RV X denote the amount of beer (in ml) in a bottle. Then $X \sim N(\mu, \sigma^2)$ with $\mu = 350$. We seek σ^2 such that

$$0.95 \leq P(X \geq 345) = P\left(\frac{X-350}{\sigma} \geq \frac{345-350}{\sigma}\right)$$

$$= P(Z \geq -5/\sigma) = P(Z \leq +5/\sigma) = \Phi(5/\sigma)$$

From the table, we find $5/\sigma \geq 1.605$, or equivalently

$$\sigma \leq \frac{5}{1.65} = 3.03$$

b) Let RV Y denote the number of bottles that do not meet the requirement. Then $Y \sim B(n, p)$ where $n = 240$ and

$$p = P(X < 345) = .05$$

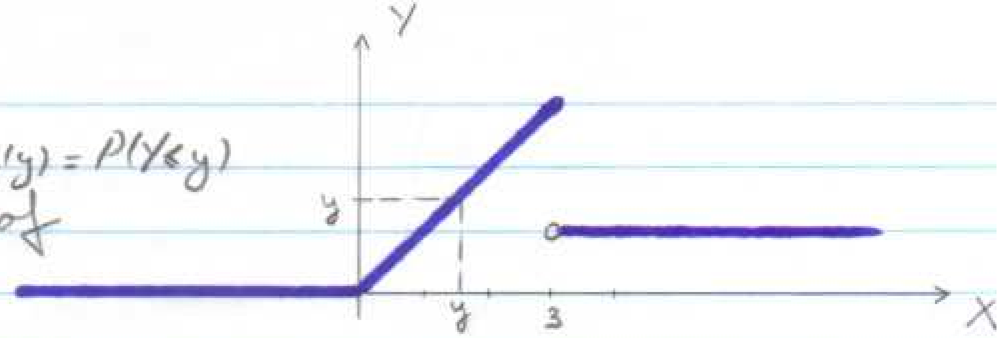
We seek

$$P(Y \geq 2) = 1 - \left[P(Y=0) + P(Y=1)\right]$$

$$= 1 - \left[\binom{240}{0} p^0 q^{240} + \binom{240}{1} p^1 q^{239}\right]$$

$$= 1 - (.95)^{240} - 240\,(.05)(.95)^{239}$$

2. a) We seek $F_Y(y) = P(Y \le y)$ using the method of distribution:



$\underline{y<0}$:

$$Y \le y \iff \emptyset = \text{impossible event}$$
$$F_Y(y) = P(\emptyset) = 0$$

$\underline{0 \le y < 1}$:

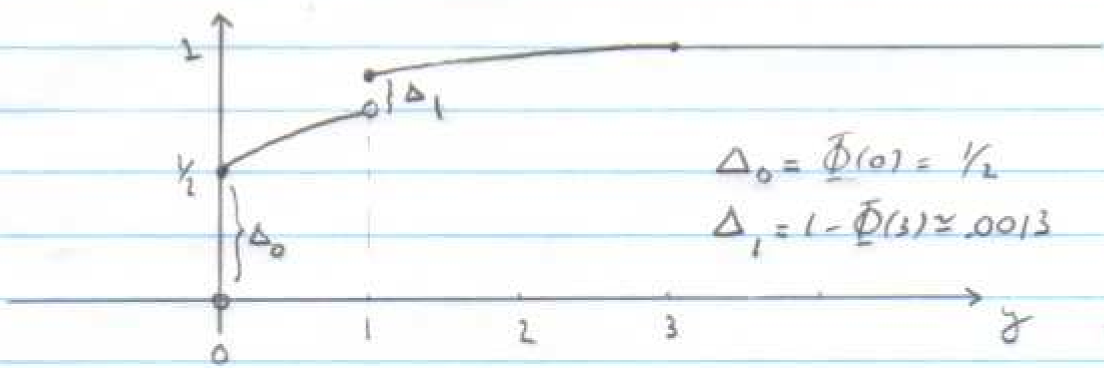
$$Y \le y \iff X \le y$$
$$F_Y(y) = P(X \le y) = \Phi(y)$$

$\underline{1 \le y < 3}$:

$$Y \le y \iff X \le y \text{ or } X > 3 \quad \text{(see Figure)}$$
$$F_Y(y) = P(X \le y \text{ or } X > 3)$$
$$= P(X \le y) + P(X > 3)$$
$$= \Phi(y) + 1 - \Phi(3)$$

$\underline{y \ge 3}$:

$$Y \le 3 \iff S = \text{certain event}$$
$$F_Y(y) = P(S) = 1$$

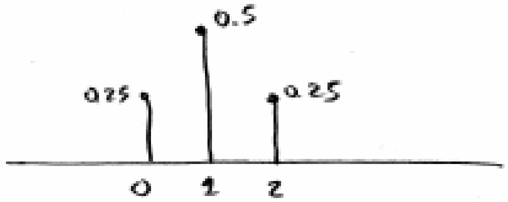


$\Delta_0 = \Phi(0) = 1/2$

$\Delta_1 = 1 - \Phi(3) \simeq .0013$

b) $$f_Y(y) = \frac{d}{dy} F_Y(y)$$
$$= \phi(y)[u(y) - u(y-2)] + \Delta_0 \delta(y) + \Delta_1 \delta(y-1)$$

Q.3



$R_x = \{0, 1, 2\}$

(a) $E(X) = 0 \cdot p(0) + 1 \cdot p(1) + 2 \cdot p(2) = 1 \times 0.5 + 2 \times 0.25 = 1 = \mu_x$

$Var(X) = E\left((X - \mu_x)^2\right) = (0-1)^2 p(0) + (1-1)^2 p(1) + (2-1)^2 p(2)$

$= 0.25 + 0.25$

$= 0.5$

(b) $\psi(w) = E\{e^{-jwX}\} = e^{-jw \cdot 0} \cdot p(0) + e^{-jw1} \cdot p(1) + e^{-jw2} p(2)$

$= 0.25 + 0.5 e^{-jw} + 0.25 e^{-j2w}$

(c) $\psi'(w) = 0.5(-j) e^{-jw} + 0.25(-2j) e^{-j2w}$

$\Rightarrow \psi'(0) = 0.5(-j) + 0.5(-j) = -j$

$\Rightarrow E(X) = (j)^1 \cdot (-j) = 1$

$\psi''(w) = 0.5(-j)^2 e^{-jw} + 0.25(-2j)^2 e^{-j2w}$

$= -0.5 e^{-jw} + e^{-j2w}$

$\Rightarrow \psi''(0) = -0.5 - 1 = -1.5$

$\Rightarrow E(X^2) = (j)^2 (-1.5) = 1.5$

So, $Var(X) = E(X^2) - E(X)^2 = 1.5 - 1^2 = 0.5$