1. a) BBBB GGGG

or GGGG BBBB

4 item 4 item
2 identical

$4! \times \frac{4!}{2!} \times 2 = 576$

b) (GGGG) BBBB

4 item

5 item
2 identical

$4! \times \frac{5!}{2} = 1440$

c) (BB) BB GGGG

1 item

7 item

$7! = 5040$

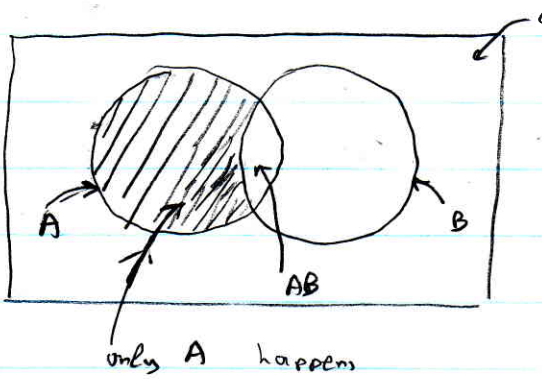
d) BGBGBGBG

or GBGBGBGB

$4! \times \frac{4!}{2} \times 2 = 576$

Question 2

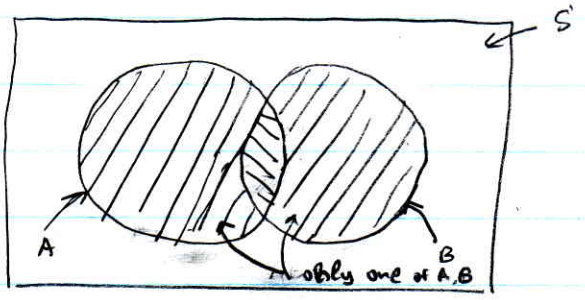
(a) Note: We ignore event C for this part.



/// only $A = AB^c$

From the diagram: $AB^c = A - AB \Rightarrow P(AB^c) = P(A) - P(AB)$

(b) Note: We ignore event C for this part



/// only one of A, B

$= AB^c \cup A^cB$

From the diagram

$P(\text{"only one of A,B"}) = P(A) + P(B) - 2P(AB)$

↳ to compensate for the double-counting of the intersection AB: (≡ region).

For parts 2(a) and (b) remember that the students were not asked to prove the relationships. They were just asked to illustrate them using Venn diagrams. A correct Venn diagram and a brief description suffice. The most important is the correct Venn diagram.

Question 2(c)

$$P(AC|B) = P(A|B)\,P(C|B)$$

$$\Rightarrow \frac{P(A \cap C \cap B)}{P(B)} = P(A|B)\,P(C|B)$$

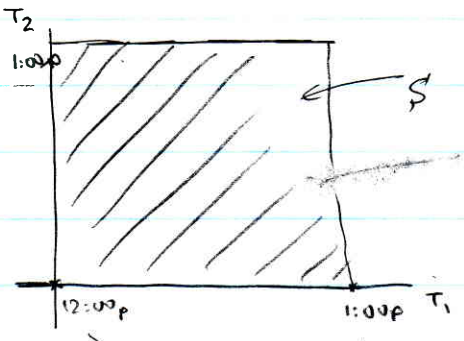
$$\Rightarrow P(A \cap C \cap B) = P(A|B)\,\underbrace{P(C|B)\,P(B)}_{P(BC)}$$

$$\Rightarrow P(A|BC)\,P(BC) = P(A|B)\,P(BC)$$

$$\Rightarrow \boxed{P(A|BC) = P(A|B)}$$

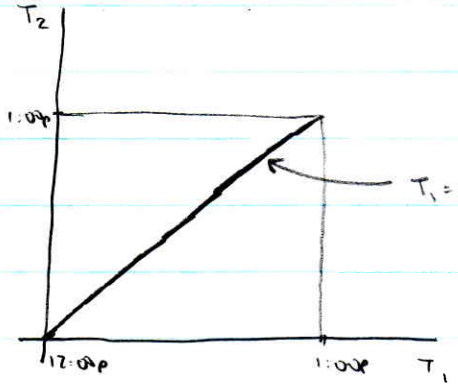
Question 3

(a)



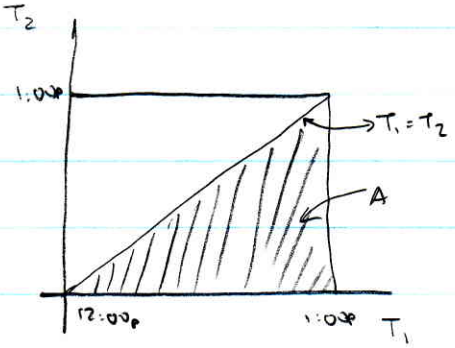
The limits are not important as long as the interval is one hour long

(b)



$P(T_1 = T_2) = 0$

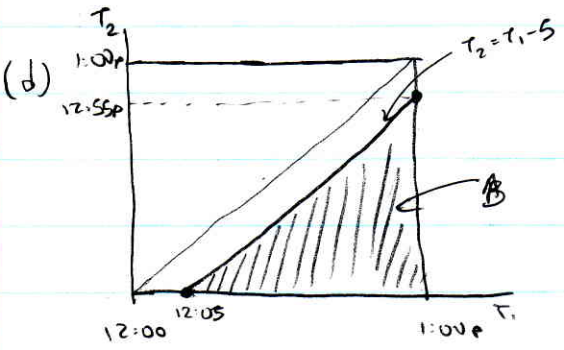
(c)



$$P(T_2 < T_1) = \frac{\text{area of } A}{\text{area of } S} = \frac{\frac{1}{2} \times 1 \times 1 \text{ hr}^2}{1 \times 1 \text{ hr}^2} = \frac{1}{2}$$

Question 3 (continued)

(d)



$$P(T_2 < T_1 - 5) = \frac{\text{area of } AB}{\text{area of } S} = \frac{\frac{1}{2} \cdot \left(\frac{55}{60}\right)\left(\frac{55}{60}\right) \text{ hr}^2}{1 \times 1 \text{ hr}^2}$$

$$\approx 0.42$$