McGill University, Faculty of Engineering
Course ECSE-305A: Probability and Random Signals I
Midterm Examination #1, Fall 2006

Date and time: Friday, October 6, 2006, 10:35 - 11:25

Examiner: Prof. B. Champagne and Y. Psaromiligkos

Instructions: This is a closed book examination: only the faculty standard calculator is allowed, NO crib sheet. Attempt all questions. **NOTE:** this exam spans 1 page

1. A debating panel of students consists of 4 boys (including John and his twin brother) and 4 girls. Find the number of different ways they can sit in a row if: 20 marks

(a) The boys and girls are each to sit together;

(b) Just the girls are to sit together;

(c) John and his brother must be seated next to each other.

(d) No two students from the same sex can be next to each other.

2. Let $(\mathcal{S}, \mathcal{F}, P)$ be a probability space and let $A, B, C \in \mathcal{F}$ be three events. 20 marks

(a) Verify using Venn diagrams that the probability that only A happens is

$$P(A) - P(AB)$$

(b) Verify using Venn diagrams that the probability that exactly *one* of the events A, B happens is

$$P(A) + P(B) - 2P(AB)$$

(c) Show that if the events A and C are *conditionally independent* given B i.e.,

$$P(AC|B) = P(A|B)P(C|B)$$

then

$$P(A|BC) = P(A|B)$$

3. A commuter train departs at a random time between 12:00pm and 1:00pm. Also, during the same one-hour period, a bus arrives at the commuter train station at a random time. A random experiment consists of recording the pair (T_1, T_2) of the train departure time and the bus arrival time. 20 marks

(a) Find the sample space of this experiment and sketch it on the (x, y)-plane.

(b) Find the probability that the bus will arrive at the station at the same time as the train leaves.

(c) Find the probability that the bus will arrive at the station before the train leaves.

(d) Find the probability that the bus will arrive at the station at least 5' before the train leaves.