1. a) $P = \dfrac{11^8}{12^8}$ $(\cong 0.50)$

b) $P = 1 - \dfrac{P(12,8)}{12^8} = 1 - \dfrac{12!/4!}{12^8}$ $(\cong 0.954)$

c) $P = \dfrac{\binom{8}{3} P(12,6)}{12^8} = \dfrac{8! \quad 12!}{3!5! \quad 6! \; 12^8}$ $(\cong 0.087)$

d) $P = \dfrac{\binom{12}{2}\binom{10}{4} \times \binom{8}{2}\binom{6}{2} \times 4 \times 3 \times 2 \times 1}{12^8}$

$= \dfrac{12! \; 8!}{2^3 \; 4! \; 6! \; 12^8}$ $(\cong 0.325)$

Problem 2

$A =$ "Read La Presse"

$B =$ "Read The Gazette"

We are given $P(A) = 0.5$, $P(B) = 0.4$, $P(A^c B^c) = 0.2$

(a) $P(AB) = P((A^c \cup B^c)^c) = 1 - P(A^c \cup B^c) = 1 - P(A^c) - P(B^c) + P(A^c B^c)$

$= 1 - (1 - 0.5) - (1 - 0.4) + 0.2$

$= 0.1$

(b) $P(AB^c) = P(A) - P(AB) = 0.5 - 0.1 = 0.4$

(c) $P(AB^c \cup A^c B) = P(AB^c) + P(A^c B)$ since AB^c, $A^c B$ are mutually exclusive

$= P(A) - P(AA) + P(B) - P(AB)$

$= 0.5 - 0.1 + 0.4 - 0.1 = 0.7$

(d) $P(AB^c \cup A^c B \mid A) = \dfrac{P((AB^c \cup A^c B)A)}{P(A)} = \dfrac{P(AB^c)}{P(A)} = \dfrac{0.4}{0.5} = 0.8$

Problem 3

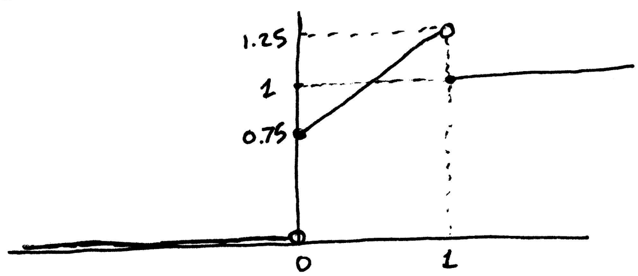
(a) We must have

$q(x) \geq 0$ ① and $q(0) + q(1) = 1$ ②

From ② we obtain $a + 2a^2 - 1 = 0 \Leftrightarrow a = 1/2$ or $a = -1$

① is satisfied only for $\boxed{a = 1/2}$

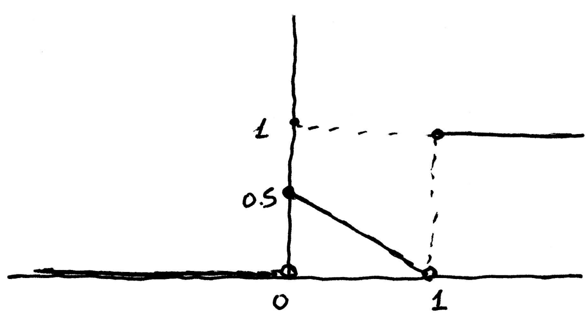
(b) $c = 0.5$, $d = 0.75$



Not a CDF

(takes values greater than 1 and is non-increasing)

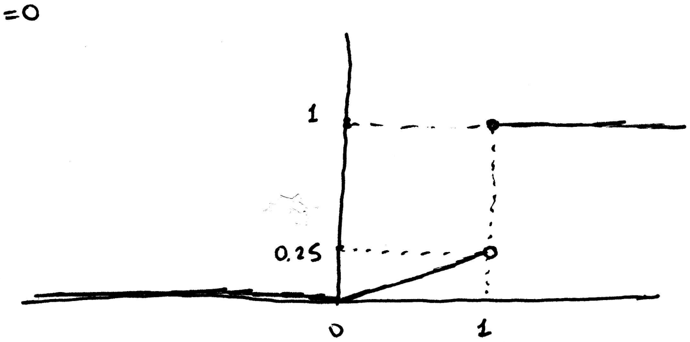
$c = -0.5$, $d = 0.5$



Not a CDF

(non-increasing)

$c = 0.25$, $d = 0$



CDF of a mixed RV

(c) We must have $c = 0$, $d = 1/2$