

**McGill University, Faculty of Engineering**  
**Course ECSE-305A: Probability and Random Signals I**  
**Midterm Examination #1, Fall 2005**

**Date and time:** Friday, October 7, 2005, 10:35 - 11:25

**Examiner:** Prof. B. Champagne and Y. Psaromiligkos

**Instructions:** This is a closed book examination: only the faculty standard calculator is allowed, NO crib sheet. Attempt all questions. **NOTE:** this exam spans 2 pages

1. Consider a group of 8 randomly selected students.

20 marks

- (a) What is the probability that none of them were born during the month of July? Assume that birth rates are approximately the same for the different months of the year.
- (b) What is the probability that at least 2 of them were born during the same month (but not necessarily in the same year)?
- (c) What is the probability that exactly 3 of them were born during the same month while the others were born in different months?
- (d) What is the probability that there are 2 months each containing the birthday of two students, and 4 months each containing the birthday of 1 student.

2. A recent study in Montreal found that 50% of the people read La Presse, 40% of the people read The Gazette, while 20% of the people do not read any of the two.

20 marks

- (a) Find the probability that a randomly selected person reads both newspapers.
- (b) Find the probability that a randomly selected person reads La Presse but not The Gazette.
- (c) Find the probability that a randomly selected person reads exactly one newspaper.
- (d) A randomly selected person reads La Presse. What is the probability that La Presse is the only newspaper that she reads?

3. Consider the following functions:

20 marks

$$q(x) = \begin{cases} a, & x = 0 \\ 2a^2, & x = 1 \\ 0, & \text{otherwise} \end{cases}$$

and

$$g(x) = \begin{cases} 0, & x < 0 \\ cx + d, & 0 \leq x < 1 \\ 1, & 1 \leq x \end{cases}$$

- (a) Find the value (or values) of  $a$  so that  $q(x)$  is a probability mass function (PMF).

- (b) For each of the following cases, sketch the graph of  $g(x)$  and indicate whether or not it is a cumulative distribution function (CDF). If it is, identify the type of the corresponding RV. Justify your answer.
- $c = 0.5, d = 0.75$
  - $c = -0.5, d = 0.5$
  - $c = 0.25, d = 0$
- (c) Find the values of  $c, d$  so that  $q(x)$  and  $g(x)$  are the PMF and the CDF, respectively, of the *same* RV.