ECSE-305 (Fall 2005)
Probability and Random Signals I

Assignment 9

November 7, 2005

Q#	Marks
1.	
2.	
3.	
4.	
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Question 1.

Let the joint probability function of joint random variables X and Y be given by

$$p(x,y)=\begin{cases}\frac{1}{25}(x^2+y^2) & if\ x=1,2\ \ y=0,1,2\\ 0 & \text{otherwise.}\end{cases}$$

Find $P(X>Y)$, $P(X+Y\le 2)$ and $P(X+Y=2)$.

Question 2.

The joint probability density function of X and Y is given by

$$f(x,y)=\frac{6}{7}(x^2+\frac{xy}{2}) \quad 0<x<1,\ 0<y<2$$

(a) Verify that this is indeed a joint density function.
(b) Compute the density function of X.
(c) Find $P(X>Y)$

Question 3.

Let X and Y have the joint probability density function

$$f(x,y)=\begin{cases}1 & if\ \ 0\le x\le 1,\ 0\le y\le 1\\ 0 & \text{elsewhere.}\end{cases}$$

Calculate $P(X+Y\le 1/2)$, $P(X-Y\le 1/2)$, $P(XY\le 1/4)$ and $P(X^2+Y^2\le 1)$.

Question 4.

A man invites his fiancée to an elegant hotel for a Sunday brunch. They decide to meet in the lobby of the hotel between 11:30 A.M. and 12 noon. If they arrive at random times during this period, what is the probability that the first to arrive has to wait at least 12 minutes?

Question 5.

The joint probability density function of X and Y is bivariate normal with $\sigma_x=\sigma_y=9$, $\mu_x=\mu_y=0$ and $\rho=0$. Find

(a) $P(X\le 6, Y\le 12)$
(b) $P(X^2+Y^2\le 36)$

Question 6.

At a certain university, the joint probability density function of X and Y, the grade average of a student in his or her freshman and senior years, respectively, is bivariate normal. From the grades of past years it is known that $\mu_X=3$, $\mu_Y=2.5$, $\sigma_X=0.5$, $\sigma_Y=0.4$ and $\rho=0.4$. Find the probability that a student with

grade average 3.5 in his or her freshman year will earn a grade average of at least 3.2 in his or her senior year.

Question 7.

An unbiased coin is flipped until the sixth head is obtained. If the third head occurs on the fifth flip, what is the probability function of the number of flips?

Question 8.

Choose a number X at random from the set of $\{1, 2, 3, 4, 5\}$. Now choose a number at random from the subset no larger than X, that is, from $\{1, \ldots, X\}$. Call this second number Y.

(a) Find the joint mass function of X and Y.
(b) Find the conditional mass function of X given that $Y = i$. Do it for $i = 1, 2, 3, 4, 5$.
(c) Are X and Y independent? Why?