ECSE-305 (Fall 2005)
Probability and Random Signals I

Assignment 8

October 31, 2005

Student Name: ID:

1. ______________________________ ____________________

Section: ______

2. ______________________________ ____________________

Section: ______

Q#	Marks
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
Total	

Question 1.

Let X be a Rayleigh random variable. Calculate its mean $E(X)$ and variance Var(X).

Question 2.

A cdf of a random variable is given by

$$F_X(x) = \begin{cases} 0 & -\infty < x < 0 \\ 4x/10 & 0 \le x < 2 \\ 1 & 2 \le x < \infty \end{cases}$$

Find its pdf and $E(X)$.

Question 3.

Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} (1/\pi)x\sin x & \text{if } 0 < x < \pi \\ 0 & \text{otherwise.} \end{cases}$$

Prove that

$$E(X^{n+1}) + (n+1)(n+2)E(X^{n-1}) = \pi^{n+1}.$$

$E(X^{n+1})$ is the (n+1)th moment of X.

Question 4.

Let X be a uniform random variable over the interval (a, b). Find the characteristic function of X.

Question 5.

Let X be a geometric random variable with parameter p. Find the characteristic function of X.

Question 6.

Let X be a gamma random variable with parameters r and λ. Derive a formula for its characteristic function $\psi(\omega)$, and use it to calculate $E(X)$ and Var(X).

Question 7.

Assume X and Y are two random variables. (X, Y) takes on three values (1, 0), (0, 1) and (1, 1) with probability 1/3 each. If $F_{X,Y}(x,y)$ is the joint cdf of X and Y, Calculate $F_{X,Y}(0,0)$, $F_{X,Y}(2,0.5)$, $F_{X,Y}(0.75,3)$ and $F_{X,Y}(1.5,1.5)$.

Question 8.

Two dices are rolled. The sum of the outcomes is denoted by X and the absolute of their difference by Y. Calculate the joint probability mass function of X and Y and the marginal probability functions of X and Y.