ECSE-305 (Fall 2005)
Probability and Random Signals I

Assignment 7

October 24, 2005

Student Name:

1. ______________________________

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Q#	Marks
1.	
2.	
3.	
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7.	
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9.	
10.	
Total	

Question 1.

The viscosity of a brand of motor oil is normal with mean 37 and standard deviation 10. What is the lowest possible viscosity for a sample that has viscosity higher than at least 90% of the other samples?

Question 2.

Let

$$\Psi(x) = \begin{cases} 2\Phi(x) - 1, x \ge 0 \\ 0, \text{otherwise} \end{cases}$$

where $\Phi(x)$ is the standard normal cdf. The function $\Psi(x)$ is called the positive normal distribution. Prove that if Z is standard normal then $|Z|$ is positive normal.

Question 3.

Suppose that lifetime of light bulbs produced by a certain company are normal random variables with mean 1000 hours and standard deviation 100 hours. Suppose that lifetimes of light bulbs produced by a second company are normal random variables with mean 900 hours and standard deviation 150 hours. Howard buys one light bulb manufactured by the first company and one by the second company. What is the probability that at least one of them lasts 980 or more hours?

Question 4.

The lifetimes of interactive computer chips produces by a certain semiconductor manufacturer are normally distributed with mean $\mu = 1.4 \times 10^6$ hours and standard deviation $\sigma = 3 \times 10^5$ hours. What is the approximate probability that a batch of 100 chips will contain at least 20 whose lifetimes are less that 1.8×10^6?

Question 5.

A point is selected at random on a line segment at length l. This point divides the line into two segments. What is the probability that none of the two segments is smaller than $l / 3$?

Question 6.

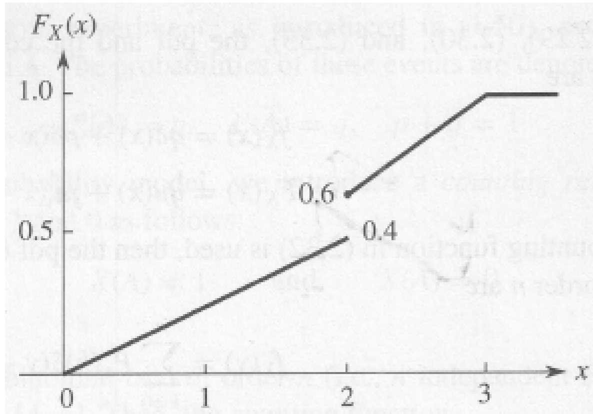
Let X, the lifetime (in years) of a radio tube, be exponentially distributed with mean $1/\lambda$. Prove that $[X]$, the integer part of X, which is the complete number of years that the tube works, is a geometric random variable.

Question 7.

For 70% of lectures, professor X arrives on time. When professor X is late, the arrival time delay is a continuous random variable uniformly distributed from 0 to 10 minutes. Yet, as soon as professor X is 5 minutes late, all the students get up and leave (It is unknown if professor X still conducts the lecture.) If a lecture starts when professor X arrives and always ends 80 minutes after the scheduled starting time, what is the pdf of T, the length of time that the students observe a lecture?

Question 8.

Suppose a random variable X has the cdf shown in the figure.



(a) Calculate and sketch the pdf of *x*.
(b) Using the pdf, find the probabilities P(X<1) and P(1<X<3).