ECSE-305 (Fall 2004)
Probability and Random Signals I

Assignment 1

September 8, 2004

Student Name: ID:

1. _____ _____

Section:

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2. _____ _____

Section:

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Q#	Marks
1.	
2.	
3.	
4.	
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10.	
Total	

Question 1.

Suppose that the loss in a certain investment, in thousand of dollars, is a continuous random variable X that has a density function of the form

$$f(x) = \begin{cases} k(2x - 3x^2) & -1 < x < 0 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Calculate the value of k.
(b) Find the probability that the loss is at most $500.

Question 2.

Let X be a continuous random variable with density and distribution functions f and F, respectively. Assuming that $\alpha \in \mathbf{R}$ is a point at which $P(X \leq \alpha) < 1$, prove that

$$h(x) = \begin{cases} f(x)/[1 - F(\alpha)] & \text{if } x \geq \alpha \\ 0 & \text{if } x \geq \alpha \end{cases}$$

is also a probability density function.

Question 3.

Let X be a continuous random variable with distribution function F and density function f. Calculate the density function of the random variable $Y = e^X$.

Question 4.

Let X be a random variable with the probability density function given by

$$f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Let

$$Y = \begin{cases} X & \text{if } X \leq 1 \\ 1/x & \text{if } X > 1 \end{cases}$$

Find the probability density function of Y.

Question 5.

Let X be a continuous random variable with the density function

$$f(x) = \begin{cases} 3e^{-x} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Using the method of transformations, find the probability density function of $Y = \log_2 X$.

Question 6.

Let *X* be a random variable with probability density function

$$f(x) = \frac{1}{2}e^{-|x|}, \quad -\infty < x < \infty$$

Calculate Var(*X*).

Question 7.

Let *X* be a random variable with density function

$$f(x) = \frac{e^{-|x|}}{2}, \quad -\infty < x < \infty$$

Find $P(-2 < X < 1)$.

Question 8.

Let *X* be a continuous random variable with distribution function *F* and density function *f*. Find the distribution function and the density function of $Y = |X|$.