ECSE-305 (Fall 2005)
Probability and Random Signals I

Assignment 5

October 11, 2005

Student Name:

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Q#	Marks
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Total	

Question 1.

Suppose that X is a discrete random variable with E(X) = 1 and E[X (X-2)] = 3. Find Var(-3X+5).

Question 2.

Suppose that two teams play a series of games that ends when one of them has won 2 games. Suppose that each game played is, independently, won by team A with probability p. Find the variance of the number of games played and show that it is maximized when $p = 1/2$.

Question 3.

Each night different meteorologists give us the probability that it will rain the next day. To judge how well these people predict, we will score each of them as follows: If a meteorologist says that it will rain with probability p, then he or she will receive a score of

$$\begin{array}{ll} 1-(1-p)^2 & \text{if it does rain} \\ 1-p^2 & \text{if it does not rain} \end{array}$$

We will then keep track of scores over a certain time span and conclude that the meteorologist with the highest average score is the best predictor of weather. Suppose now that a given meteorologist is aware of this and so wants to maximize his or her expected score. If this person truly believes that it will rain tomorrow with probability p^*, what value of p should he or she assert so as to maximize the expected score?

Question 4.

The simplest error detection scheme used in data communication is *parity-checking*. Usually messages sent consist of characters, each character consisting of a number of bits (a *bit* is the smallest unit of information and is either 1 or 0). In parity-checking, a 1 or 0 is appended to the end of each character at the transmitter to make the total number of 1's even. The receiver checks the number of 1's in every character received, and if the result is odd it signals an error. Suppose that each bit is received correctly with probability 0.999, independently of other bits (parity bit can also be in error). What is the probability that a 7-bit character is received in error, but the error is not detected by the parity check?

Question 5.

A computer network consists of several stations connected by various media (usually cables). There are certain instances when no message is being transmitted. At such "suitable instances," each station will send a message with probability p independently of the other stations. However, if two or more stations send messages, a collision will corrupt the messages and they will be discarded. These messages will be retransmitted until they reach their destination. Suppose that the network consists of N stations.

(a) What is the probability that at a "suitable instance" a message is initiated by one of the stations and will go through without a collision?
(b) Show that, to maximize the probability of a message going through with no collision, exactly one message, on average, should be initiated at each "suitable instance."
(c) Find the limit of the maximum probability obtained in (b) as the number of station of the network grows to ∞ .
(d) How many times, on average, would a certain station transmit and retransmit a message until it reaches its destination?

Question 6.

In a forest, the number of trees that grow in a region of area R has a Poisson distribution with mean λR, where λ is a given positive number.

(a) Find the probability that the distance from a certain tree to the nearest tree is more than d.
(b) Find the probability that the distance from a certain tree to the nth nearest tree is more that d.

Question 7.

The department of mathematics of a state university has 26 faculty members. For $i = 0, 1, 2, 3$, find p_i, the probability that i of them were born on Canada Day.

(a) Using the binomial distribution;
(b) Using the Poisson distribution.

Assume that the birth rates are constant through out the year and that each year has 365 days.