ECSE-305 (Fall 2005)
Probability and Random Signals I

Assignment 4

September 30, 2005

Q#	Marks
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
Total	

Student Name: ID:

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Question 1.

A random variable X is symmetric about 0 if for all $x \in \mathbf{R}$,

$$P(X \geq x) = P(X \leq -x)$$

Prove that if X is symmetric about 0, then for all $t > 0$ its distribution function F satisfies the following relations:

(a) $P(|X| \leq t) = 2F(t) - 1$

(b) $P(|X| > t) = 2[1 - F(t)]$

(c) $P(X = t) = F(t) + F(-t) - 1$

Question 2.

Let the time until a new car breaks down be denoted by X, and let

$$Y = \begin{cases} X & \text{if } X \leq 5 \\ 5 & \text{if } X > 5 \end{cases}$$

Then Y is the life of the car, if it lasts less than 5 years, and is 5 if it lasts longer than 5 years. Calculate the distribution function of Y in terms of F, the distribution function of X.

Question 3.

In successive rolls of a fair die, let X be the number of rolls until the first 6 appears. Determine the probability mass function and the distribution function of X.

Question 4.

Let X be a random point selected from the interval (0, 1). Calculate F, the distribution function of $Y = X/(1+X)$, and sketch its graph.

Question 5.

A box contains 20 fuses, of which five are defective. What is the expected number of defective items among three fuses selected randomly?

Question 6.

If X is a random number selected from the first 10 positive integers, what is $E[X(11-X)]$?

Question 7.

An urn contains five balls, two of which are marked \$1, two \$5, and one \$15. A game is played by paying \$10 for winning the sum of the amount marked on two balls selected randomly from the urn. Is this a fair game?

Question 8.

(a) Show that

$$p(n) = \frac{1}{n(n+1)} \quad n \geq 1 ,$$

is a probability mass function

(b) Let X be a random variable with probability mass function p given in part (a); find $E(X)$.