

ECSE-305 (Fall 2005)
Probability and Random Signals I

Assignment 3

September 23, 2005

Student Name:

1. _____

ID:

Section:

2. _____

Section:

Q#	Marks
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
Total	

Question 1.

A number is selected at random from the set $\{1, 2, \dots, 10,000\}$ and is observed to be odd. What is the probability that it is

- (a) divisible by 3;
- (b) divisible by neither 3 nor 5?

Question 2.

From an ordinary deck of 52 cards, cards are drawn one by one, at random and without replacement. What is the probability that the fourth heart is drawn on the tenth draw?

Question 3.

From families with three children, a child is selected at random and found to be a girl. What is the probability that she has an older sister? Assume that both sexes are equiprobable.

Question 4.

Among 1000 fuses there can a maximum of 5 defective fuses. More specifically, there are 0 defective fuses with probability $1/6$, exactly 1 defective fuse with probability $1/6$, exactly 2 defective fuses with probability $1/6$, exactly 3 defective fuses with probability $1/6$, exactly 4 defective fuses with probability $1/6$ or exactly 5 defective fuses with probability $1/6$.

We select 100 fuses at random and we see that none is defective. What is the probability of no defective fuses at all in the original 1000 fuses?

Question 5.

A high school student is anxiously waiting to receive a mail telling her whether she has been accepted to a certain college. She estimated that the conditional probabilities, given that she is accepted and that she is rejected, of receiving notification on each day of next week are as follows:

Day	$P(\text{mail} \mid \text{accepted})$	$P(\text{mail} \mid \text{rejected})$
Monday	0.15	0.05
Tuesday	0.20	0.10
Wednesday	0.25	0.10
Thursday	0.15	0.15
Friday	0.10	0.20

She estimated that her probability of being accepted is 0.6.

- (a) What is the probability that mail is received on Monday?
- (b) What is the conditional probability that mail is received on Tuesday given that it is not received on Monday?
- (c) If there is no mail through Wednesday, what is the conditional probability that she will be accepted?
- (d) What is the conditional probability that she will be accepted if mail comes on Thursday?

Question 6.

A fair coin is tossed n times. Show that the event “at least two heads” and “one or two tails” are independent if $n = 3$ but dependent if $n = 4$.

Question 7.

Die A has 4 red and 2 white faces, whereas die B has 2 red and 4 white faces. A fair coin is flipped once. If it lands on heads, die A is rolled; if it lands on tails then die B is rolled.

- (a) Show that the probability of red at any throw is $1/2$.
- (b) If the first two throws result in red, what is the probability of red at the third throw?
- (c) If red turns up at the first throws, what is the probability that it is die A that is being used?

Question 8.

A pair of dice is rolled 50 times. What is the probability of obtaining double six at least three times?