ECSE-305 (Fall 2005)
Probability and Random Signals I

Assignment 11

December 2, 2005

Q#	Marks
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
Total	

Student Name: ID:

1. ______________________ ______________

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Section: ____

Question 1.

Let X be a random variable with mean μ. Show that if $E[(X-\mu)^{2n}] < \infty$, then for $\alpha > 0$

$$P(|X-\mu| \geq \alpha) \leq \frac{1}{\alpha^{2n}} E[(X-\mu)^{2n}].$$

Question 2.

Let X_1, X_2, … be a sequence of independent, identically distributed random variables with mean $\mu < \infty$. Let $S_n = X_1 + X_2 + ... + X_n$, and $\overline{X_n} = \frac{S_n}{n}$. Show that S_n grows at rate n. That is,

$$\lim_{n \to \infty} P(n(\mu - \varepsilon) \leq S_n \leq n(\mu + \varepsilon)) = 1.$$

Question 3.

A biologist wants to estimate L, the average life expectancy of a certain type of insect. To do so, he takes a sample of n insects, measures their lifetimes and sample-averages them. If he believes that the lifetimes of these insects are independent random variables with variance 1.5 days^2, how large a sample should he choose to be 98% sure that his average is accurate within ± 4.8 hours? *Hint:* Use the Central Limit Theorem.

Question 4.

Using the Central Limit Theorem find the probability that the average of 150 random points from the interval (0, 1) is within 0.02 of the midpoint of the interval.

Question 5.

Suppose that, whenever invited to a party, the probability that a person attends with his or her guest is 1/3, attends alone is 1/3, and does not attend is 1/3. A company has invited all 300 of its employees and their guests to a certain Christmas party. Using the Central Limit Theorem find the probability that at least 320 people will attend.

Question 6.

The random variable Δ is uniform in (0, T). Find the autocorrelation function $R(t_1, t_2)$ of the random process

$$X(t) = U(t-\Delta),$$

where $U(t)$ is the unit step function.

Question 7.

Consider the process

$$X(t) = A\cos(\omega t + \Theta)$$

where A is a random amplitude exponentially distributed with parameter λ, and Θ is a random phase uniformly distributed between $-\pi$ and π and independent of A. Show that $X(t)$ is wide sense stationary.

Question 8.

A wide sense stationary process $x(t)$ with autocorrelation function $R_X(\tau) = 100e^{-100|\tau|}$ is the input to an RC filter with impulse response

$$h(t) = \begin{cases} e^{-t/RC} & t \geq 0 \\ 0 & \text{Otherwise.} \end{cases}$$

The filter output process has average power $E[y^2(t)] = 100$.

(a) Find the output autocorrelation $R_Y(\tau)$.

(b) What is the value of RC?