

ECSE-305 (Fall 2004)
Probability and Random Signals I

Assignment 1

September 8, 2004

Student Name:

1. _____

ID:

Section:

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Q#	Marks
1.	
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Question 1.

Let X and Y be random variables having joint density function

$$f(x, y) = \begin{cases} 2 & \text{for } 0 \leq x \leq 1, 0 \leq y \leq x \\ 0 & \text{otherwise.} \end{cases}$$

Define $Z = 2X + Y$. Determine density and distribution function of Z .

Question 2.

Let X and Y be independent random variables with common probability density function

$$f(x) = \begin{cases} e^{-x} & \text{if } x > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Find the joint probability density function $U = X + Y$ and $V = e^X$.

Question 3.

Let X and Y have joint probability density function

$$f(x, y) = \begin{cases} \frac{3}{2} (x^2 + y^2) & \text{if } 0 \leq x \leq 1, -1 \leq y \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find $E(X^2 + Y^2)$ and $E(X + e^{-Y})$.

Question 4.

From an ordinary deck of 52 cards, four cards are drawn at random and without replacement. Let X and Y be the number of hearts and the number of spades drawn, respectively.

- Find the joint probability function of X and Y .
- Calculate the average (expected) sum of the drawn hearts and spades in these four cards, i.e. $E(X + Y)$.

Question 5.

A fair die is thrown n times. What is the covariance and correlation coefficient of the number of 1's and the number of 6's obtained?

Question 6.

(a) Let X_1, X_2, \dots, X_n be independent exponential random variable with means $1/\lambda_1, 1/\lambda_2, \dots, 1/\lambda_n$, respectively. Find the probability distribution of $X = \min(X_1, X_2, \dots, X_n)$.

(b) An item has n parts, each with an exponentially distributed lifetime with mean $1/\lambda$. If the failure of one part makes the item fail, what is the average lifetime of the item?

Question 7.

Let X , Y and Z be positive independent random variables with the identical probability density function e^{-x} for $x > 0$. Find the joint probability function of $U = X + Y$, $V = X + Z$ and $W = Y + Z$.

Question 8.

Let X_1, X_2, \dots, X_n be n independent random numbers from the interval $(0, 1)$. Find $E(\max_{1 \leq i \leq n} X_i)$,

$E(\min_{1 \leq i \leq n} X_i)$ and $E(\sum_{i=1}^n X_i)$.