ECSE-305 (Fall 2005)
Probability and Random Signals I

Assignment 1

September 9, 2005

Q#	Marks
1.	
2.	
3.	
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Question 1.

Let E, F, and G be three events. Determine which of the following statements are correct and which are incorrect. Justify your answers.

(a) $(E-(E\bigcap F))\bigcup F = E\bigcup F$.
(b) $(F^c \bigcap G)\bigcup(E^c \bigcap G) = G\bigcap(F\bigcup E)^c$.
(c) $(E\bigcup F)^c \bigcap G = E^c \bigcap F^c \bigcap G$.

Question 2.

Let A and B be two events. Prove the following relations by the elementwise method.

(a) $(A - A\bigcap B)\bigcup B = A\bigcup B$.
(b) $(A\bigcup B)-(A\bigcap B) = (A\bigcap B^c)\bigcup(A^c \bigcap B)$.

Question 3.

Use Venn diagram to illustrate that for any A, B and C,

(a) $(A\bigcap B)^c = A^c \bigcup B^c$.
(b) $(A\bigcap B)-C = (A-C)\bigcap(B-C)$.

Question 4.

Let $\{A_n\}_{n=1}^{\infty}$ be a sequence of events. Prove that for every event B,
$B\bigcup(\bigcap_{i=1}^{\infty} A_i) = \bigcap_{i=1}^{\infty}(B\bigcup A_i)$.

Question 5.

How many different arrangements are there of the letters A, B, C, D, E, F for which

(a) A and B are next to each other;
(b) A is before B;
(c) A is before B and B is before C;
(d) A is before B and C is before D;
(e) A and B are next to each other and C and D are also next to each other;
(f) E is not last in the line.

Question 6.

In how many ways can n identical balls be distributed into r urns so that the ith urn contains at least m_i balls, for each i = 1, …, r? Assume that $n \geq \sum_{i=1}^{r} m_i$.

Question 7.

In a formal banquet at United Nations, 20 presidents and their vice presidents are seated at random at a round table. In how many ways they can sit so that all presidents are sitting next to their vice presidents? Suppose that of these 20 presidents, ten presidents wear glasses. In how many ways they can sit so that all presidents with glasses are sitting next to their vice presidents? Note that when people are sitting around a round table, only their seats relative to each other matter. The exact position of a person is not important.

Question 8.

Show that for $n > 0$,

$$\sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0$$

HINT: Use the binomial theorem.