ECSE-305, Winter 2009
Probability and Random Signals I
Assignment #9

Posted: Thursday, March 26, 2009.

Due: Thursday, April 2, 2009, 2h30pm.

Student #1:

Name:____________________________

ID:________________________________

Student #2:

Name:____________________________

ID:________________________________

Question	Marks
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10.	
Total	

1. Let X be a Normal $N(0, \sigma^2)$ random variable, and let $Y = X^2$. Are X and Y independent? Are they correlated?

2. Assume that the value of a resistor is a random variable R whose value in Ohm is uniformly distributed on $(4, 6)$. To this resistor, we apply a voltage V that is random and distributed according to a Normal $N(10, 1)$ density. Find the mean and the variance of the current flowing through the resistor. Assume that R is independent of V.

3. Let the joint probability mass function of random variables X, Y and Z be given by

$$p(x, y, z) = \begin{cases} cxyz, & \text{if } (x, y, z) \in \mathcal{R}_X \times \mathcal{R}_Y \times \mathcal{R}_Z \\ 0, & \text{otherwise} \end{cases}$$

where $\mathcal{R}_X = \{4, 5\}$, $\mathcal{R}_Y = \{1, 2, 3\}$ and $\mathcal{R}_Z = \{1, 2\}$

   (a) Find the constant c?

   (b) Find the marginal PMF of the RVs X, Y and Z.

   (c) Find the joint marginal PMF of the pair X, Y, Y, Z and X, Z, i.e. find $p_{XY}(x, y)$, $p_{YZ}(y, z)$ and $p_{XZ}(x, z)$.

4. Let the joint probability density function of X, Y and Z be given by

$$f(x, y, z) = \begin{cases} cxyz, & \text{if } (x, y, z) \in (0, 1)^3 \\ 0, & \text{otherwise} \end{cases}$$

   (a) Find the constant c.

   (b) Find the marginal PDF of the RVs X, Y and Z.

   (c) Find $\rho(X, Y)$, $\rho(X, Z)$ and $\rho(Y, Z)$.

5. From the set of families with two children a *family* is selected at random. Let $X_1 = 1$ if the first child of the family is a girl; $X_2 = 1$ if the second child of the family is a girl; and $X_3 = 1$ if the family has exactly one boy. For $i = 1, 2, 3$, let $X_i = 0$ in other cases. Determine if X_1, X_2 and X_3 are independent. Assume that in a family the probability that a child is a girl is independent of the gender of the other children and is $1/2$.

6. Suppose that on average, a postoffice handles 20000 letters a day with a standard deviation of 1000 letters. What can be said about the probability that on any given day, this postoffice will handle between 19000 and 21000 letters?

7. Find the variance of a sum of n randomly and independently selected points from the interval $(0, 1)$.

8. (a) Let X be a binomial RV with parameters n and p, i.e. $X \sim B(n, p)$. Derive a compact expression for the characteristic function of X.

   (b) Let Y be a sum of k independent random variables X_i, i.e.

   $$Y = X_1 + X_2 + \cdots + X_k$$

   Assuming that RVs $X_i \sim B(n_i, p)$, show that Y is also binomial, i.e. $Y \sim B(n', p')$. Find the corresponding parameters n' and p'.

   (c) Consider the transmission of $k = 512$ binary packets, each consisting of $n = 1024$ bits. Assume that each bit transmission is independent with probability of error $p = 10^{-2}$. Using the central limit theorem, evaluate the probability that the total number of bits received in error is less than 128.

9. In a survey of voting intentions for the next Quebec Government general election, how large should be the sample size n to ensure that the estimated proportion of voters for a given party is within $\pm 1\%$ of the true voting intention at least 19 times out of 20. (Hint: assume n large and apply central limit theorem).

10. Generalize Theorem 12.4 in the class notes to the case of uncorrelated RVs with different means $\mu_i = E(X_i)$, but common variance $\sigma^2 = Var(X_i)$:

    (a) Restate the Theorem accordingly.

    (b) Prove it (you will need to modify the proof on p. 306).