# ECSE-305, Winter 2009
# Probability and Random Signals I
# Assignment #8

**Posted:** Thursday, March 19, 2009.

**Due:** Thursday, March 26, 2009, 2h30pm.

**Important notes:**

- Assignments without this cover page will be discarded.

**Student #1:**

Name:_____

ID:_____

**Student #2:**

Name:_____

ID:_____

| Question | Marks |
|----------|-------|
| 1. | |
| 2. | |
| 3. | |
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| 5. | |
| 6. | |
| 7. | |
| 8. | |
| 9. | |
| 10. | |
| Total | |

1. The joint PDF of random variables $X$ and $Y$ is given by

$$f(x,y) = \begin{cases} cx^k y^l, & 0 \le x \le 1, \, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

   where $k$ and $l$ are positive integers.

   (a) Find an expression for the constant $c$ in terms of $k$ and $l$

   (b) Find a necessary condition on the values of $k$ and $l$ such that $P(Y > X) > 1/2$.

   (c) Find the marginal PDFs of $X$ and $Y$.

2. Using an appropriate software package (e.g. Matlab), sketch the 3D graph of the joint normal PDF $N(\mu_X, \mu_Y, \rho, \sigma_X^2, \sigma_Y^2)$ for the following parameter values (note: hand-drawing will not be accepted):

   (a) $\mu_X = \mu_Y = 0$, $\sigma_X = \sigma_Y = 1$ and $\rho = 0$,;

   (b) $\mu_X = 1, \mu_Y = 1/2$, $\sigma_X = \sigma_Y = 1$ and $\rho = 0$,;

   (c) $\mu_X = \mu_Y = 0$, $\sigma_X = 1, \sigma_Y = 2$ and $\rho = 0$,;

   (d) $\mu_X = \mu_Y = 0$, $\sigma_X = \sigma_Y = 1$ and $\rho = 0.5$.

3. Prove Theorem 9.4 in the class notes.

4. From an ordinary deck of cards, 8 cards are drawn at random and without replacement. Let $X$ and $Y$ be the number of clubs and spades respectively. Determine whether or not $X$ and $Y$ are independent.

5. Let $X$ and $Y$ be uniformly distributed over the region $D = [a, b] \times [c, d]$ where $a$, $b$, $c$ and $d$ are unspecified constants.

   (a) Find the joint PDF $f(x, y)$ and sketch it in 3 dimensions.

   (b) Find and sketch the joint CDF of $X$ and $Y$.

   (c) Determine whether or not $X$ and $Y$ are independent.

6. Let $X$ and $Y$ be two independent, zero-mean Gaussian RV with variance $\sigma^2$. Define the polar coordinates

$$R = \sqrt{X^2 + Y^2} \in [0, \infty), \qquad \Theta = \angle(X, Y) \in [0, 2\pi).$$

- Find the joint PDF of $R$ and $\Theta$.

- Find the marginal PDFs of $R$ and $\Theta$.

7. Let $X$ and $Y$ be two independent exponential random variables with parameter $\lambda$. Obtain an expression for the CDF of $Z = \max(X, Y)$.

8. Let the joint probability density function of $X$ and $Y$ be bivariate normal. For what values of $\alpha$ is the variance of $\alpha X + Y$ minimum.

9. Let the joint probability density function of $X$ and $Y$ be given by

$$f(x, y) = \begin{cases} 2e^{-(x+2y)} & \text{if } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find $E(X^2Y)$.

10. If $\Theta$ is a random number from the interval $[0, 2\pi]$, are the dependent random variables $X = \sin\Theta$ and $Y = \cos\Theta$ correlated? Why or why not?