ECSE-305, Winter 2009
Probability and Random Signals I
Assignment #7

Posted: Tuesday, March 12, 2009.

Due: Tuesday, March 19, 2009, 2h30pm.

Important notes:

- Assignments without this cover page will be discarded.

Student #1:

Name:__________________________

ID:______________________________

Student #2:

Name:__________________________

ID:______________________________

Question	Marks
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9.	
10.	
Total	

1. Suppose that for any positive integer n, the nth moment of a random variable X, is given by $E(X^n) = (n+1)!\,2^n$. Obtain a closed form expression for $\psi(\omega)$, the characteristic function of X.

2. Let X be a a continuous RV with the probability density function $f(x) = 6x(1-x)$, if $0 \le x \le 1$ and 0 elsewhere.
(a) Find the characteristic function of X.
(b) Using the characteristic function, find $E(X)$.

3. Using the moment-generating function of a poisson random variable X with parameter λ, find $E(X)$ and $Var(X)$.

4. Let $\psi_X(\omega) = 1/(1+j\omega)$ be the moment-generating function of a random variable X. Find the moment-generating function of the random variable $Y = 2X + 1$.

5. Let the joint probability mass function of two jointly distributed discrete RVs X and Y be

$$p(i,j) = \begin{cases} k(i+j) & \text{if } i,j \in \{1,2,3\} \\ 0 & \text{elsewhere.} \end{cases}$$

   (a) Find the value of the constant k.

   (b) Calculate $P(X = 1, Y < 3)$, $P(X = 1, Y \le 3)$, $P(X = 2)$, $P(X < Y)$, $P(X \le Y)$.

6. Let the joint PMF of discrete RVs X and Y be

$$p(i,j) = \begin{cases} k(i^2+j^2) & \text{if } (i,j) \in \{(1,1),(1,3),(2,3)\} \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find the value of the constant k.
(b) Find the marginal PMFs of X and Y.

7. The joint probability density function of random variables X and Y is given by

$$f(x,y) = \begin{cases} 2 & \text{if } 0 \le y \le x \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Calculate the marginal PDFs of X and Y.
(b) Calculate $P(X < 1/2)$, $P(X < 2Y)$, and $P(X = Y)$.

8. On a line segment AB of length l, two points C and D are placed at random and independently. What is the probability that C is closer to D than to A?

9. Two RVs X and Y are jointly uniform on $[0, 1]^2$. Calculate the probability $P(Y \leq X \text{ and } X^2 + Y^2 \leq 1)$.