ECSE-305, Winter 2009
Probability and Random Signals I
Assignment #6

Posted: Tuesday, March 3, 2009.

Due: Tuesday, March 10, 2009, 2h30pm (in class).

Important notes:

- Assignments without this cover page will be discarded.

Student #1:

Name:__________________________

ID:_____________________________

Student #2:

Name:__________________________

ID:_____________________________

Question	Marks
1.	
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9.	
10.	
Total	

1. The PMF of a discrete RV X is given by

$$p(x) = 1/15, \quad x \in \{1, 2, 3, 4, 5\}$$

and $p(x) = 0$ otherwise.

   (a) Find $E(X)$, $E(X^2)$ and $Var(X)$.

   (b) Let $Y = X(6 - X)$. Find the PMF of Y, say $p_Y(y)$.

   (c) Find $E(Y)$ using two different approaches

2. Suppose that on average, there is one typographical error in every 5 pages of a book. Assuming that the number of such errors on a single page is a Poisson random variable, what is probability of at leats one error on a specific page of the book.

3. Let X be a random number from $[0, 1]$. Find the probability mass function of $Y = \lfloor nX \rfloor$, the greatest integer less than or equal to nX.

4. A right triangle has a hypotenuse of length 9. If the PDF of one side's length is given by

$$f(x) = \begin{cases} x/6 & \text{if } 2 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

what is the expected value of the length of the other side?

5. Let X be a random variable with probability density function

$$f(x) = \frac{1}{2}e^{-|x|}, \;\; x \in R.$$

Calculate $Var(X)$.

6. A random variable X has the density function

$$f(x) = \begin{cases} 3e^{-3x} & \text{if } 0 \le x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Calculate $E(e^X)$.

7. Let $X \sim N(\mu, \sigma^2)$. Find the CDF of $Y = |X - \mu|$ and its expected value, $E(Y)$.

8. In the nineteenth century, a Belgian astronomer named Adolph Quetelet observed that (what we know today as) the normal distribution applied to many human phenomena. In particular, he gathered information on the chest sizes of more than five thousand Scottish soldiers and found that the data traced a bell-shaped curve centered on the average chest size, about 40 inches, with inflection points at 37.5 and 42.5 inches, approximately. (There is a lot of information on this available on the web). What is the probability that of 50 randomly selected Scottish soldiers (in those days), 10 had a chest size of at least 40 inches.

9. The scores of a math test given to several thousands grade-V high school students across Quebec is normally distributed with mean 75% and standard deviation 10%.

   (a) What should be the score of a student to place him/her among the top 10%?

   (b) Repeat the above for top 5% and top 2%.

10. Let X, the life time (in years) of a radio tube, be exponentially distributed with mean $1/\lambda$. Prove that $Y = \lfloor X \rfloor$, the integer part of X, which is the complete number of years that the tube works, is a geometric random variable.

11. Let X be a standard normal RV. Show that $Y = X^2$ follows a gamma distribution and find its parameters.

12. The CDF of a RV X is given as follows:

$$F(x) = \begin{cases} 0, & x < -1 \\ \Phi(x), & -1 \leq x < 1 \\ 1, & x \geq 1 \end{cases}$$

   (a) Show that X is a mixed RV. That is, identify the 4 components entering the definition of a mixed RV as seen in class, namely: α, β, $F_d(x)$ and $F_c(x)$.

   (b) Find the generalized PDF of X and sketch it.

   (c) Find the following probabilities: $P(X < 0)$, $P(X \leq 0)$, $P(X < 1)$ and $P(X \leq 1)$.

   (d) Find the expected value of X and also show that $Var(X) < 1$.