ECSE-305, Winter 2009
Probability and Random Signals I
Assignment #5

Posted: Tuesday, Feb. 10, 2009.

Due: Tuesday, Feb. 17, 2009, 2h30pm (in class).

Important notes:

- Do NOT use the assignment box in Trottier anymore.
- Assignments without this cover page will be discarded.

Student #1:

Name:________________________

ID:________________________

Student #2:

Name:________________________

ID:________________________

Question	Marks
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Total	

1. A certain basketball player makes a foul shot with probability 0.45. What is the probability that (a) his first basket occurs later than the sixth shot; (b) his first basket occur on his fourth shot and his second basket occur before his eight shot?

2. Suppose that jury members decide independently, and that each with probability p $(0 < p < 1)$ makes the correct decision. If the decision of the majority is final, which is preferable, a three-person jury or a single juror. (Hint: you want to maximize the probability of a correct decision; the answer depends on p).

3. The ECE Department of a certain University has 45 faculty members. For $i = 0, 1, 2, 3$, find p_i, the probability that i of them were born on January 1st

   (a) using the Binomial distribution;

   (b) using the Poisson distribution.

4. A mother ask her daughters to clean the dishes after diner. Since she does not specify which of the three daughters is to do the job, each girl tosses a coin to determine the odd person, who must then clean the dishes. In the case that all three get heads or tails, they continue tossing until they reach a decision. Let p be the probability of heads and $q = 1 - p$.

   (a) Find the probability that they reach a decision in less than n tosses.

   (b) If $p = 1/2$, what is the minimum number of tosses required to reach a decision with probability 0.95?

5. Negative binomial RVs are generalizations of geometric RVs. Consider a sequence of identical, independent Bernouilli trials, each with probability of success $0 < p < 1$. Let X be the number of trials until the rth success, where $r \in \mathbb{N}$ is a given integer. RV X is called a *negative binomial* with parameters (r, p).

   (a) Find the range $\mathcal{R}_X$ of X, i.e. the set of possible values of X.

   (b) Derive an expression for the PMF of X.

(c) Bill and Monica are playing a series of backgammon games until one of them wins five games. Suppose that the games are independent and the probability that Bill wins a game is 0.42. Find the probability that the series and in 8 games.

6. The CDF of a random variable X is:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4}x & 0 \leq x < 4 \\ 1 & x \geq 4 \end{cases}$$

(a) Explain why X is a continuous RV.
(b) Find the PDF of X.
(b) Find the following probabilities using the PDF of X:

$$\begin{array}{lll} P(X \geq 5) & P(X < 0) & P(X \leq 0) \\ P(\frac{1}{4} \leq X < 1) & P(\frac{1}{4} \leq X \leq 1) & P(X > \frac{1}{2}) \end{array}$$

7. Let X denote the life time of a radio, in years, manufactured by a certain company. The density function of X is given by

$$f(x) = \begin{cases} \frac{1}{15}e^{-x/15} & 0 \leq x < \infty \\ 0 & \text{elsewhere.} \end{cases}$$

What is the probability that, of eight such radios, at least four last more than 15 years?

8. Let X be a random variable with the density function

$$f(x) = \frac{1}{\pi(1+x^2)}, \qquad -\infty < x < \infty$$

Find the density function of $Z = \arctan X$.

9. Let X be a random variable with the probability density function given by

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Let

$$Y = \begin{cases} X & \text{if } X \leq 1 \\ \frac{1}{X} & \text{if } X > 1. \end{cases}$$

Find the probability density function of Y.