ECSE-305, Winter 2009
Probability and Random Signals I
Assignment #4

Posted: Tuesday, Feb. 03, 2009.

Due: Tuesday, Feb. 10, 2009, 2h30pm (in class).

Important notes:

- Do NOT use the assignment box in Trottier anymore.
- Assignments without this cover page will be discarded.

Student #1:

Name:________________________

ID:________________________

Student #2:

Name:________________________

ID:________________________

Question	Marks
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Total	

1. The CDF of a random variable X is given by:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4}(x+1) & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

   (a) Sketch $F(x)$ and indicate the type of random variable X.
   (b) Find the following probabilities:

$$\begin{array}{lll} P(X \geq 5), & P(X < 0), & P(X \leq 0), \\ P(\frac{1}{4} \leq X < 1), & P(\frac{1}{4} \leq X \leq 1), & P(X > \frac{1}{2}). \end{array}$$

2. The CDF of a random variable X is:

$$F(x) = \begin{cases} 0 & x < -2 \\ 1/4 & -2 \leq x < 0 \\ 1/2 & 0 \leq x < 1 \\ 3/4 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

   (a) Explain why X is a discrete RV.
   (b) Find the PMF of X.
   (c) Find the PMF of X^2.

3. In successive rolls of a fair die, let X be the number of rolls until the first 6 appears. Determine the PMF and the CDF of X.

4. Let X be a non-negative integer-valued discrete RV. Show that

$$E(X) = \sum_{k=0}^{\infty} P(X > k)$$

5. The random variable X takes the values $0, 1, \ldots$ with $P(X = k) = p_k$. Let $Y = (X-1)u(X-1)$ where $u(.)$ denotes the unit step function. Express $E(Y)$ and $E(Y^2)$ in terms of $E(X)$ and $E(X^2)$.

6. A newly married couple decides to continue having children until they have one of each sex. If the events of having a boy and a girl are independent and equiprobable, how many children should this couple expect?

7. A drunken man has n keys, one of which opens the door to his office. He tries the keys at random, one by one, and independently. Compute the mean and the variance of the number of trials required to open the door if the wrong keys (a) are not eliminated; (b) are eliminated.