ECSE-305, Winter 2009
Probability and Random Signals I
Assignment #3

Posted: Tuesday, Jan. 27, 2009.

Due: Tuesday, Feb. 3, 2009, 2h30pm (in class).

Important notes:

- Do NOT use the assignment box in Trottier anymore.
- Assignment without this cover page will not be marked.

Student #1:

Name:____________________________

ID:____________________________

Student #2:

Name:____________________________

ID:____________________________

Question	Marks
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Total	

1. A man randomly chooses one of the six parks in his neighborhood everyday and goes there for hiking. He was seen in one of these parks, Westmount Park, once during the last ten days. What is the probability that during this period, he has hiked in this park two or more times?

2. Cards are drawn at random from an ordinary deck of 52, one-by-one and without replacement. Let A denote the event that no heart is drawn before the ace of spades is drawn.

   (a) Find $P(A)$ by first viewing A as a union of 39 mutually exclusive events and then using the law of total probability.

   (b) Repeat but this time, view A as a union of $\binom{52}{14}$ mutually exclusive events.

3. In a series of independent games, the winning number of the nth game, where $n = 1, 2, \ldots,$ is a number selected at random from the set of integers $\{1, 2, 3, \ldots, n+2\}$. Don bets on 1 in each game and says that he will quit as soon as he wins. What is the probability that he has to play indefinitely?

4. Prove the following properties of conditional probabilities:

   (a) For any events A, B and C such that $P(BC) \neq 0$, we have

   $$P(AB|C) = P(A|BC)P(B|C)$$

   (b) Let $B_1, B_2, \ldots, B_n$ form a partition of the sample space. Then

   $$P(A|C) = \sum_{i=1}^{n} P(A|B_iC)P(B_i|C)$$

5. From families with three children, a child is selected at random and found to be a girl. What is the probability that she has an older sister? (Hint: Note that the selected child can be the oldest, the middle or the youngest child; then use the result of problem 4.b)

6. In a study it was discovered that 25% of the paintings of a certain gallery are not original. a collector in 15% of the cases makes a mistake in judging if a painting is authentic or a copy. If she buys a piece thinking that it is original, what is the probability that it is not.

7. A fair coin is flipped indefinitely. What is the probability of (a) at least one head in the first n flips; (b) exactly k heads in the first n flips; (c) getting heads in all of the flips indefinitely.

8. How many equations are needed to establish the mutual independence of n events ($n > 1$)?

9. Hemophilia is a hereditary disease. If a mother has it, then with probability $1/2$, any of her sons independently will inherit it. If the mother does not have it, none of the sons becomes hemophilic. Julie is the mother of two sons, and from her family' medical history it is known that, with the probability $1/4$, she is hemophilic. What is the probability that:

   (a) her first son is hemophilic;

   (b) her second son is hemophilic;

   (c) none of her sons are hemophilic?