ECSE-305, Winter 2009
Probability and Random Signals I
Assignment #2

Posted: Tuesday, Jan. 20, 2009

Due: Tuesday, Jan. 27, 2009, NO later than 2h30pm (please use the assignment box)

Student #1:

Name:_____

ID:_____

Student #1:

Name:_____

ID:_____

Question	Marks
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10.	
Total	

1. An integrated circuit factory has three machines X, Y, and Z. Suppose that we test at random one integrated circuit produced by each machine. Either a circuit passes the test (p) or fails the test (f). An observation is a sequence of three test results corresponding to the circuits from machine X, Y, and Z, respectively. For example ppf is the observation that the circuits from X and Y pass the test and the circuit from Z fails the test.

   (a) What are the elements of the sample space of this experiment?

   (b) What are the elements of the set:
      $Z_F =\{$ circuit from Z fails the test $\}$
      $X_P =\{$ circuit from X passes the test $\}$

   (c) Are Z_F and X_P mutually exclusive?

   (d) What is the probability of $Z_F \cup X_P$?

   (e) What are the elements of the sets:
      $C=\{$ more than one circuit pass the test $\}$
      $D=\{$ at least two circuits fail the test $\}$

   (f) Are C and D mutually exclusive?

   (g) What is the probability of $C \cup D$?

2. Let $(S, \mathcal{F}, P)$ be a probability space and $A, B, C \in \mathcal{F}$.

   (a) Show that exactly two of the events A, B, C happen with probability
   $$P(AB) + P(AC) + P(BC) - 3P(ABC)$$

   (b) Which of the following statements is true?
      S1: If $P(A) + P(B) + P(C) = 1$ then the events A, B, C are mutually exclusive.
      S2: If $P(A \cup B \cup C) = 1$ then the events A, B, C are mutually exclusive.

      Prove the statements that are true; for the false statements provide a counterexample.

3. In a specified 24-hour period beginning at midnight (time 0), a student wakes up at a random time T_1 and goes to sleep at some later random

time T_2;

(a) Find the sample space and sketch it on the x-y plane if the outcome of this experiment consists of the pair (T_1, T_2);

(b) Sketch the region on the plane corresponding to the event A: "student is awake at 9 o'clock", and calculate the probability of A;

(c) Sketch the region on the plane corresponding to the event B: "student is asleep more time than he is awake", and calculate the probability of B;

(d) Sketch the region corresponding to $A^c \cap B$, describe the corresponding event in words and calculate the probability of $A^c \cap B$.

4. A number x is selected at random in the interval $[-1, 1]$. Let the events $A = \{ x < 0 \}$, $B = \{ |x - 0.5| < 1 \}$, and $C = \{ x > 0.75 \}$.

(a) Find the probabilities of B, $A \cap B$, and $A \cap C$;

(b) Find the probabilities of $A \cup B$, $A \cup C$.

5. A fair coin is tossed n times. Calculate the probability of getting no successive heads.

6. We place at random n particles in $m > n$ boxes. Find the probability p that the particles will be found in n preselected boxes (one in each box) in the following cases:

(a) M-B (Maxwell-Boltzmann): the particles are distinct;

(b) B-E (Bose-Einstein): the particles cannot be distinguished;

(c) F-D (Fermi-Dirac): the particles cannot be distinguished, and at most one particle is allowed in a box.

7. Suppose that 20 sticks are broken, each into one long and one short part. By pairing them randomly, the 40 parts are then used to make 20 new sticks. What is the probability that long parts are all paired with short ones? What is the probability that the new sticks are exactly the same as the old ones?

8. From a shuffled deck of 52 cards you choose randomly 14 cards. What is the probability that you choose two hearts, three diamonds, four clubs and five spades?

9. Place 10 different photographs into 6 different envelopes. Find the probability that no envelope is empty.