ECSE-305, Winter 2009
Probability and Random Signals I
Assignment #1

Posted: Tuesday, Jan. 13, 2009

Due: Tuesday, Jan. 20, 2009, NO later than 2h30pm (please use the assignment box)

Student #1:

Name:_____

ID:_____

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Name:_____

ID:_____

Question	Marks
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Total	

1. Let $A = \{1, 2, \ldots, 8, 9\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 3, 5, 7, 9\}$, $D = \{3, 4, 5\}$ and $E = \{3, 5\}$. Which of the above sets, represented by X, satisfy the given conditions?

   (a) X and B are disjoint;

   (b) $X \subseteq D$ but $X \nsubseteq B$;

   (c) $X \subseteq A$ but $X \nsubseteq C$;

   (d) $X \subseteq C$ but $X \nsubseteq A$.

2. State whether the following sets are finite or infinite sets. If infinite, specify if they are countable or uncountable.

   (a) $A = \{\text{all prime numbers}\}$

   (b) $B = \{x \in \mathbb{R} : 0 < x < 10\}$

   (c) $C = \{x \in \mathbb{Q} : 0 < x < 10\}$

   (d) $D = \{x \in \mathbb{N} : 0 < x < 10\}$

   (e) $E = \{x \in \mathbb{N} : x^2 = 10\}$

3. Show that each of the sets $A \cup B$ and B can be represented as a union of mutually exclusive sets as follows:

$$A \cup B = A \cup (B - A) \quad \text{and} \quad B = (A \cap B) \cup (A^c \cap B)$$

   Illustrate each situation by means of a Venn diagram.

4. Consider the following sequences (i.e. indexed families) of intervals of the real axis, where the index $i \in \mathbb{N}$:

$$A_i = (-\frac{1}{i}, \frac{1}{i}), \quad B_i = [-\frac{1}{i}, \frac{1}{i}], \quad C_i = (0, 1 - \frac{1}{i}), \quad D_i = [0, 1 - \frac{1}{i}]$$

   In each case, indicate whether the sequence is increasing or decreasing and find the corresponding limit as $i \to \infty$.

5. There are 3 bus lines linking city A to city B and 2 bus lines linking city B to city C (each bus line is available in both directions). Find the number of ways a person can travel by bus:

   (a) from A to C by way of B;

(b) round trip from A to C to A by way of B;

(c) round-trip from A to C to A by way of B, without using a bus line more than once;

(d) illustrate the situation in (c) by means of a tree diagram

6. A debating team of students consists of 3 boys and 3 girls. Find the number of different ways they can sit in a row if:

   (a) there are no restrictions;

   (b) the boys and girls are each to sit together;

   (c) just the girls are to sit together;

   (d) Explain how the above solutions would need to be modified if the team members were to sit in a circle.

7. A class contains 9 boys and 3 girls. Find the number of ways a teacher can select a committee of 4 where:

   (a) there are no restrictions;

   (b) there must be 2 boys and 2 girls;

   (c) exactly 1 girl;

   (d) at least 1 girl.

8. Find the number of distinguishable permutations that can be formed from all the letters of the word "COMMITTEE". How many of these begin by the letter C and end with the letter E?

9. 10 persons are waiting at an airport counter, of whom 5 are from Canada and five are from France. How many different line-ups can be formed so that no two persons from the same country are next to each other?

10. Prove Theorem 2.10 in the notes using mathematical induction. Can you provide a simple "combinatorial" explanation for the appearance of the binomial coefficient $\binom{n}{i}$ in the polynomial expansion of $(x+y)^n$?