Ans

10.1) $y(t) = \sin(\theta t + w), \quad t \in \mathbb{R}.$

$w \sim U[0, 2\pi] \Rightarrow w = \begin{cases} \frac{1}{2\pi} & \text{for } 0 < w < 2\pi \\ 0 & \text{o/w} \end{cases}$

(a) $E[y(t)] = \int_{-\infty}^{\infty} y(t) \cdot f_w(w)\, dw$

$= \int_0^{2\pi} \sin(\theta t + w) \cdot \frac{1}{2\pi}\, dw$

$= \frac{1}{2\pi} \left[\cos(\theta t + w)\right]_{w=2\pi}^{0}$

$= \frac{1}{2\pi} \left[\cos(\theta t) - \cos(\theta t + 2\pi)\right]$

$= \frac{1}{2\pi} \left[\cos(\theta t) - \cos(\theta t)\right]$

$= 0.$

(b) $E[y(t)] = E[y(s)] = 0$, from part(a).

$\Rightarrow C(t,s) = E[y(t) \cdot y(s)]$

$= \int_{-\infty}^{\infty} \sin(\theta t + w) \cdot \sin(\theta s + w) \cdot f_w(w)\, dw$

$= \int_0^{2\pi} \sin(\theta t + w) \cdot \sin(\theta s + w) \cdot \frac{1}{2\pi}\, dw$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{\cos(\theta t + w - \theta s - w) + \cos(\theta t + w + \theta s + w)}{2}\, dw$$

as $\sin A . \sin B = \dfrac{\cos(A-B) + \cos(A+B)}{2}$

$$= \frac{1}{4\pi}\int_0^{2\pi} \cos(\theta t - \theta s) + \cos(\theta t + \theta s + 2w)\, dw$$

$$= \frac{1}{4\pi}\left[\cos(\theta t - \theta s).w + \frac{\sin(\theta t + \theta s + 2w)}{2}\right]_{w=0}^{2\pi}$$

$$= \frac{1}{4\pi}\left[2\pi.\cos(\theta t - \theta s) + \frac{1}{2}\sin(\theta t + \theta s + 4\pi) - 0 - \frac{1}{2}\sin(\theta t + \theta s)\right]$$

$$= \frac{1}{4\pi}\left[2\pi.\cos(\theta t - \theta s) + \cancel{\frac{1}{2}\sin(\theta t + \theta s)} - \cancel{\frac{1}{2}.\sin(\theta t + \theta s)}\right]$$

$$= \frac{1}{4\pi}.2\pi\cos(\theta t - \theta s)$$

$$= \frac{1}{2}.\cos\{\theta(t-s)\}$$

$$= \frac{1}{2}.\cos(\theta \tau), \text{ where } \tau = t - s.$$

$\therefore$ $y(t)$ is WSS.

Ans 10·2) $\quad B(t) = I(t) - K(t)$.

(a) $m^B(t) = E[B(t)]$

$= E[I(t)] - E[K(t)]$

$= m^I - m^K$.

$R^B(t+\tau, t) = E[B(t+\tau) \cdot B(t)]$

$= E[I(t+\tau) \cdot I(t)] - E[I(t+\tau) \cdot K(t)]$

$- E[K(t+\tau) \cdot I(t)] + E[K(t+\tau) \cdot K(t)]$

$= R^I(\tau) - E[I(t+\tau)] \cdot E[K(t)]$

$- E[K(t+\tau)] \cdot E[I(t)] + R^K(\tau)$,

as $I(t)$ and $K(t)$ are independant

$= R^I(\tau) - m^I \cdot m^K - m^K \cdot m^I + R^K(\tau)$

$= R^I(\tau) + R^K(\tau) - 2 \cdot m^I \cdot m^K$.

$\therefore$ $B(t)$ is WSS, as its mean is constant and R is a function of τ.

(b) $R^D(t+\tau, t) = E[D(t+\tau) \cdot D(t)]$

$$= E[s(t+\tau) \cdot B(t+\tau) \cdot s(t) \cdot B(t)]$$

$$= E[s(t+\tau) \cdot s(t)] \cdot E[B(t+\tau) \cdot B(t)]$$

as $s(t)$ and $B(t)$ are independent

$$= R^s(\tau) \cdot R^B(\tau).$$

$\therefore$ D is t-shift invariant.