ECSE 304-305B Assignment 3 Fall 2008

Return by noon, Thursday, 2nd October

Question 3.1

The diagram on the following page gives the abstracted Markov chain representation of a collection of hurricane random walk (HRW) events in the Atlantic and the Gulf of Mexico in 2004.

Assume that with probability 1 on Day 1 a hurricane is generated off the African coast at location A, and that (as shown in the diagram) on Day 2 with probability $\frac{1}{2}$ it arrives at the North Atlantic (NA) location and with probability $\frac{1}{2}$ at the Mid-Atlantic (MA) location.

Further assume that on Days 3, 4 and 5 the hurricane makes the successive Markov random transitions with the transition (i.e. conditional) properties indicated on the diagram. Assume that the $\frac{1}{3}$ probability path from the Central States (CS) location leads to a location North Carolina (NC) which is not displayed. Further assume that with probability 1 a hurricane at NC, SEC (South East Coast) or NA goes to the location A on the next day.

(i) Given that $P(A) = 1$, find the conditional probabilities:

(a) $P(CS|MA)$, (c) $P(CS|A)$, where NF denotes North Florida.

Do this by using the Markov property together with the conditional version of the Total Probability Theorem, namely, $P(A|C) = P(A|B, C)P(B|C) \quad + \quad P(A|B^c, C)P(B^c|C)$, (two element partition case: $B \cup B^c$). Do this at each stage working forwards or backwards in time.

(ii) Given that $P(A) = 1$, find the conditional probabilities:

(a) $P(MA|CS)$, (b) $P(CA|SEC)$.

(iii) Take the 11 component vector $[NC, SEC, CS, NF, CG, FK, SG, CA, MA, NA, A]^T$ to denote the state vector of the system. Find the transition matrix of the HRW model. Assume that on Day 1: $P(SG) = \frac{1}{4}, P(CA) = \frac{3}{4}$; then use the transition matrix to find the hurricane probability distribution on Day 3.

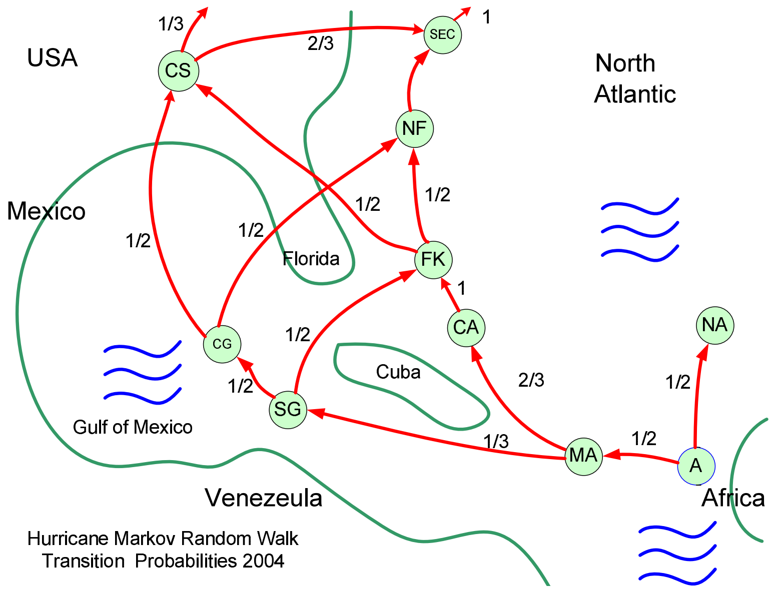

Figure 3.1: Hurricane Markov Model

Question 3.2

You have three boxes, each with one drawer on each of two sides, A and B respectively. Each drawer contains a coin. One box has a gold coin on both sides (**GG**), one a silver coin on both sides (**SS**), and the third a gold coin on one side and a sliver coin on the other (**SG**).

You choose a box at random (i.e. with probability one third for each box), open one drawer, and find a gold coin. What is the probability that the coin in the other drawer of this box is silver?

(i) Before you do part (ii), think about this problem for not more than 30secs, and then write down your estimate of the probability. You will not be marked right or wrong on your answer, but you should not change it after you have written it down. (You need to make a guess this part to get credit for part (ii).)

(ii) Find the required probability using Bayes' Theorem.

Question 3.3

A web page search algorithm has two independent stages: at Stage 1, the algorithm makes repeated independent attempts at steps $n = 1,\ 2, ..,$ to connect with page W_1. The probability of success is $p > 0$ and of failure $1 - p$. Once a success occurs, the algorithm passes to Stage 2, where it makes repeated independent attempts at steps $n = 1,\ 2, ..,$ to connect with page W_2. The probability of success is $q > 0$ and of failure $1 - q$. The algorithm halts when it connects to page W_2.

(a) Find the probability the algorithm halts after making exactly n steps of Stage 1 and then making exactly m steps of Stage 2.

(b) Find the probability that the algorithm has a success in Stage 1 at any instant over the infinite range $1, 2, ...$ and then halts after making exactly m steps in Stage 2

(c) Take $p = 1/3$ and $q = 1/4$. Find the probability that the algorithm makes an odd number $2n + 1, n = 0, 1, ...,$ of steps at Stage 1 and then halts at any unspecified iteration in the infinite range $m = 1, 2, ...$ at Stage 2

Question 3.4

An Ehrenfurst molecular Markov chain model E has four states: x_1, x_2, x_3 x_4 , corresponding to two urns (L, R) containing $x_1 = (3, 0)$, $x_2 = (2, 1)$, $x_3 = (1, 2)$ and $x_4 = (0, 3)$ particles, respectively.

(i) Find the transition matrix T for the Markov chain E.

(ii) At the initial instant $k = 0$, the occupancy probability vector $P_0(x_1) = \frac{1}{2} = P_0(x_4)$, $P_0(x_2) = P_0(x_3) = 0$. Find the occupancy probability vector at the instant $k = 3$, namely:

$$P_3 = \begin{pmatrix} P_3(x_1) \\ P_3(x_2) \\ P_3(x_3) \\ P_3(x_4) \end{pmatrix}$$

(iii) Find a possible steady state occupancy probability for the Markov chain E.