ECSE 304-305B   Assignment 2   Fall 2008

Return by noon, Thursday, 25th September

Question 2.1 (a) Consider the *multinomial distribution with parameters* N *= 4,* r *=2* for the random labelling of objects as Black or White (which is a *Binomial distribution* since r=2). The probability that an element of the set of four objects will be labelled Black is p and White is $1 - p$.

Write down the formula from the Notes (webct Number 5) for the probability of the event where m elements are labelled Black and $4 - m$ are labelled White. Treating p as an indeterminate (i.e. an algebraic variable), expand the probabilites for all possible values of m and sum the resulting polynomials. Explain the answer you obtain.

(b) Let a coin have a probability $\frac{3}{4}$ of coming down Heads in a toss and $\frac{1}{4}$ Tails. Generalize the method in the example below to this biased coin case so as to find the probability of getting two or more heads in five tosses. (This is again an example of a *multinomial distribution with parameter* r *=2*.)

Express your answer in terms of fractions and then evaluate it to three decimal places.

Example: Find the probability of getting four or more heads in six tosses of an unbiased coin.

Solution: $P(4 \text{ or more heads in 6 tosses}) = P(4) + P(5) + P(6)$ where,

$$P(k)\text{=}P(k \text{ heads in 6 tosses})\text{=}\begin{pmatrix} 6 \\ k \end{pmatrix} /2^6$$

so $P(4$ or more heads in 6 tosses$)$=$(15 + 6 + 1)/2^6$=11/32.

Question 2.2 All of the following parts are for Loto 6-15 as a modification of the Loto game on pages 3 and 4, of the Notes: Counting Probabilities (webct Number 5).

(a) What is the probability of getting exactly three numbers correct on one ticket showing 6 distinct numbers in a draw of 6 numbers without replacement (i.e. a draw of six distinct numbers)?

(b) What is the probability of getting exactly one or two numbers correct on one ticket? Give the answer as a fraction and correct to three decimal places.

(c) You buy two tickets which turn out to have no numbers in common. What is the probability of getting exactly two numbers correct on each of the tickets in one draw?

Question 2.3 The Partition Function of Statisical Mechanics is the ratio of an exponential to the sum of exponentials (each corresponding to an energy level) as given in the last section of the Lecture 5: Counting Probabilites of the Notes.

In a specific application of the Partition Function, a substance in a vessel has $n = 2^{10}$ particles at a temperature $\mu = 1/50$, in appropriate units. There are four energy levels $e_i = i^2$, for $i = 1, 2, 3, 4$.
(a) Find an expression for the number of particles in each energy level in the most likely configuration (subject to the number and energy constraints (1) and (2)) by applying the formulas for each n_i in the notes.
(b) Give a formula for the energy per particle in the most likely configuration (subject to (1) and (2)) by using the formula for $\frac{E}{N}$ in the Notes.
Question 2.4 A total of n balls are sequentially and randomly chose, without replacement, from an urn containing r red and b blue balls ($n \leq r + b$). Given that k of the n balls are blue, what is the conditional probability that the first ball chosen is blue?

Hint: Let B_f be the event that the first ball chosen is blue and let B_k be the event that a total of k blue balls are chosen.

Then using the defintion of conditional probability, obtain:

$$P(B_f|B_k) = \frac{P(B_k|B_f)P(B_f)}{P(B_k)}$$

Next find $P(B_k)$ by using C_k^b, C_{n-k}^r and C_n^{b+r}; similarly find $P(B_k|B_f)$ by using C_{k-1}^{b-1}, C_{n-k}^r and $C_{(n-1)}^{b+r-1}$.
Verify you answer by showing that for both $r = 4, b = 2, n = 3, k = 2$ and for $r = 5, b = 3, n = 3, k = 2$, the conditional probability is $\frac{2}{3}$.