ECSE 304-305B   Assignment 1   Fall 2008

Return by noon, Thursday, 18th September

Question 1.1   Assume the failure probabilities of a set of three components are such that: $P[(F_2 \cap F_3) \cap F_1] = 0.004$; $P[(F_2 \cap F_3) \cap \overline{F_1}] = 0.023$ and $P[(\overline{F_2} \cup \overline{F_3}) \cap \overline{F_1}] = 0.011$, where $\overline{F_1}$, etc, denotes complement. Find $P[F_2 \cap F_3]$ and $P[F_1]$. Can one find separately $P[F_2]$ or $P[F_3]$. ? Answer Yes or No as appropriate in each case.

Hint: Use the Inclusion - Exclusion relation for probabilities.

Question 1.2   In a branching experiment BE, a coin is such that $Prob(H) = \frac{1}{4}$, $Prob(T) = \frac{3}{4}, (H = head, T = tail)$. The coin is repeatedly tossed until a head is obtained and at that instant BE stops. Distinct tosses are independent, i.e. the probability of a specified head or tail pair at distinct instants is the product of the individual probabilities.

(a) A sample point (i.e. an outcome or singleton event) for BE consists of a sequence (possibly empty) of tails T and a terminating head H. (i) Are the outcomes $\{H, TH, T^2H, ..., T^{N-1}H, ...\}$ disjoint events? Answer Yes or No. (ii) Is the union of this set of events equal to set of all possible events? Give a reason for your answer.

(b) Use Axiom III to derive a simple expression for the probability that BE stops before or at the N^{th} toss, i.e. at an outcome $T, TH, ..., T^{n-1}H, 1 \leq n \leq N, N < \infty$.

(c) Show whether or not the elements of the set of sample points (i.e. set of singleton events) $\{H, TH, ..., T^{N-1}H, ...\}$ are pairwise independent, i.e. show whether $P(E \cap F) = P(E).P(F)$ for sample points E, F.

(d) Find the probability that the experiment BE stops (i) after an odd number of tosses, (ii) after an even number of tosses. What is the sum of the probablities in (i) and (ii)?

Question 1.3   In a specified 24-hour period, a student wakes up at a time T_1 and goes to sleep at a (not necessarily strictly) later time $T_2 \geq T_1$.

(a) Find the sample space for this experiment where the outcome is taken to be the pair (T_1, T_2).

(b) Specify the region A of the plane corresponding to the event "student is awake at 9am."

(c) Specify and sketch the set B in the plane corresponding to the event "student is asleep more time than the student is awake."

(d) Sketch the region corresponding to the event $D = A^c \cap B$ and describe the corresponding event in words. Assuming the Equiprobability Principle holds for the outcomes of the experiment, find the probability of the event D.

Question 1.4

Assume that the probabilities $P(A), P(B), P(C)$ of the events A, B, C in the probability space S are given by the limits of the corresponding relative frequency functions, along an infinite run of experiments, where the relative frequencies are given in terms of indicator functions as in Lecture 3.

Prove Propositions 1, 2 and 3 of Lecture 3 purely by use of these relative frequency definitions of probability.

Question 1.5 (From Chapter 1, Question 26,page 25, SG.) For a Democratic candidate to win an election, she must win districts I, II and III. Polls have shown that the probability of wining I and III is 0.55, losing II but not I is 0.35, and losing II and III but not I is 0.15. Find the probability that this candidate will win all three districts. (Draw a Venn diagram.)