ECSE 304-305B Solution to Assignment 3 Fall 2008

Question 3.1

(i) Given $P(A) = 1$, we have

a)

$$P(CS|MA) = P(CS|CG)P(CG|SG)P(SG|MA) + P(CS|FK)P(FK|SG)P(SG|MA)$$
$$+P(CS|FK)P(FK|CA)P(CA|MA) = (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3})$$
$$+(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3}) + (\frac{1}{2} \times 1 \times \frac{2}{3}) = \frac{1}{2}.$$

b) By using part a) we have

$$P(CS|A) = P(CS|MA)P(MA|A) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

ii)

a) By using part i) we have

$$P(MA|CS) = \frac{P(MA \cap CS)}{P(CS)} = \frac{P(CS|MA)P(MA)}{P(CS)} = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{4}} = 1.$$

b) By using the hurricane Markov model we have

$$P(CA) = P(CA|MA)P(MA|A)P(A) = \frac{1}{3},$$

$$P(SEC|CA) = P(SEC|NF)P(NF|FK)P(FK|CA)+P(SEC|CS)P(CS|FK)P(FK|CA) = \frac{5}{6},$$

$$P(SEC) = \frac{5}{12}.$$

Therefore

$$P(CA|SEC) = \frac{P(SEC|CA)P(CA)}{P(SEC)} = \frac{2}{3}.$$

iii)

$*$	NC	SEC	CS	NF	CG	FK	SG	CA	MA	NA	A
NC	0	0	$1/3$	0	0	0	0	0	0	0	0
SEC	0	0	$2/3$	1	0	0	0	0	0	0	0
CS	0	0	0	0	$1/2$	$1/2$	0	0	0	0	0
NF	0	0	0	0	$1/2$	$1/2$	0	0	0	0	0
CG	0	0	0	0	0	0	$1/2$	0	0	0	0
FK	0	0	0	0	0	0	$1/2$	1	0	0	0
SG	0	0	0	0	0	0	0	0	$1/3$	0	0
CA	0	0	0	0	0	0	0	0	$2/3$	0	0
MA	0	0	0	0	0	0	0	0	0	0	$1/2$
NA	0	0	0	0	0	0	0	0	0	0	$1/2$
A	1	1	0	0	0	0	0	0	0	1	0

By using the transition matrix, the hurricane probability distribution on Day 3 is

$$[0\ 0\ 1/2\ 1/2\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T.$$

Question 3.2

There is only one box with one gold and one silver, so $P(A) = 1/3$.

Also by using Bayes's Theorem we have:

$$P(A) = \frac{P(S|G)}{P(G)} = \frac{P(\text{first gold and second silver})}{P(\text{first gold})} = \frac{1/3 \times 1/2}{1/3 + (1/3 \times 1/2)} = \frac{1}{3}.$$

Question 3.3

$$P\ (\text{success } n \text{ by steps in stage 1 and } m \text{ steps in stage 2})$$

$$= (1-p)^{n-1} \times p \times (1-q)^{m-1} \times q.$$

(b)

$$P\ (\text{any steps in stage 1 in infinite range and } m \text{ steps in stage 2})$$

$$= \sum_{n=1}^{\infty} (1-p)^{n-1} \times p \times (1-q)^{m-1} \times q.$$

(c)

$$P\,(\text{makes in odd step in stage 1 and any instant in stage 2})$$
$$= (\sum_{n=0}^{\infty} (1-p)^{2n} \times p) \times 1 = \frac{p}{1-(1-p)^2} = \frac{1}{2-p} = \frac{3}{5}.$$

Question 3.4

a)

$$P(X_2|X_1) = 1,$$
$$P(X_1|X_2) = 1/3,$$
$$P(X_3|X_2) = 2/3,$$
$$P(X_2|X_3) = 2/3,$$
$$P(X_4|X_3) = 1/3,$$
$$P(X_3|X_4) = 1.$$

Therefore,

$$T = \begin{pmatrix} 0 & 1/3 & 0 & 0 \\ 1 & 0 & 2/3 & 0 \\ 0 & 2/3 & 0 & 1 \\ 0 & 0 & 1/3 & 0 \end{pmatrix}.$$

2)

$$P_3 = T^3 \begin{bmatrix} 1/2 \\ 0 \\ 0 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 1/9 \\ 7/18 \\ 7/18 \\ 1/9 \end{bmatrix}.$$

3)

$$[T - I] \times \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = 0 \quad X_1 + X_2 + X_3 + X_4 = 1.$$

Therefore,

steady state = $\begin{bmatrix} 1/8 & 3/8 & 3/8 & 1/8 \end{bmatrix}$.