ECSE 304-305 Assignment 6 Winter 2008

Return by noon, 30th October

Question 6.1

A scalar random variable X is sent through a *multiplicative* coding channel which generates the output $Z = XY$. The independent positive scalar random variables X and Y have the densities $\{f_X(x) = x^{-2}, 1 \leq x < \infty,\}$ and $\{f_Y(y) = 2y^{-3}\ ; 1 \leq y < \infty\}$, respectively.

(a) Find the densities of the random variables: $X' = log\, X,\ 1 \leq X < \infty$, and $Y' = log\, Y,\ 1 \leq Y < \infty$, on $R_+ = [0, \infty)$.

(b) Find the characteristic functions of the densities of X', Y' and $Z' = log\, Z$.

(c) Give the probability density of the coded version Z of the signal X.

Question 6.2

Find (a) the mean value μ, and (b) the variance σ^2 of an RV X with the *Laplace density*

$$f_X(x) = \frac{1}{2b} e^{-2|x-m|/2b},$$

where b and m are real constants, $b > 0$ and $-\infty < m < \infty$.

Find the corresponding characteristic function $\Phi_X(\omega)$ and verify the values found above for $\mu,\ \sigma^2$ by use of the Moment Theorem.

Question 6.3

(a) The exponential random variable Z has the density $f_Z(\cdot) = \{5\mu e^{-5\mu z}, z \in R_+, \mu > 0\}$. Find the characteristic function $\Phi_Z(\omega), \omega \in R$.

(b) Let the random variable W be defined by $W = 3(Z_1) - 3(Z_2)$, where Z_1 and Z_2 are independent identically distributed exponential random variables with parameter λ. Find $\Phi_W(\omega), \omega \in R$.

(c) Using the one-to-one relation of characteristic functions and densities and pa rt (b), find the density of W. (Hint: check Question 2.)

Question 6.4

X is a scalar random variable distributed $N(\mu, \sigma^2) = N(1, 4)$, $\sigma > 0$.

Let $Z = g(X) = \frac{X^2}{2}$. Find the density $f_Z(\cdot)$ of Z for $z > 0$.

Question 6.5

Many people believed (before October, 2008) that the daily change of price of a company's stock on the stock market is a random variable with mean 0 and variance σ^2 . That is, if Y_n represents the price of the stock on the nth day, then

$$Y_n = Y_{n-1} + X_n \qquad n \geq 1$$

where $X_1, X_2,...$ are independent and identically distributed random variables with mean 0 and variance σ^2. Accepting this, suppose that the stock's price today is 100. If $\sigma^2 = 1$, use the CLT to estimate the probability that the stock's price will exceed 105 after 10 days.