ECSE 304-305B Assignment 5 Fall 2008

Return by noon, 23rd October

Question 5.1

A positive scalar random variable X with a density is such that $EX = \mu < \infty$, $EX^2 = \infty$.

(a) Using whichever of the Markov or the Chebyshev inequalities is applicable, estimate the probability $P(X^2 \geq \alpha^2)$, $\alpha > 0$.

(b) Is the distribution function $F_X(x) = P(X \leq x)$, $x \in I\!R$, in the case under consideration a continuous function of x? Explain your answer.

(c) Does there exist a real number $\gamma > 0$ such that $e^{-\gamma} = P(X \leq \gamma)$ and if so why?

Question 5.2 (SG p 182)

What are the expected number, the variance and the standard deviation (= the square root of the variance) of the number of spades in a poker hand? (A poker hand is a set of five cards that are randomly selected (i.e. the EPP applies) from an ordinary deck of 52 cards.) Give your answer to three decimal places.

Question 5.3

Let X be a continuous random variable (i.e. a not a discrete random random variable, hence it takes an uncountable set of values) whose probability distribution function has the density

$$f(x) = 6x(1 - x), \qquad 0 < x < 1.$$

What is the probability that X takes a value within two standard deviations of the mean? That is to say, what is the probability that X is less than or equal to the mean plus two standard deviations but greater than or equal to the mean minus two standard deviations?

Question 5.4

Each of an i.i.d. sequence of random variables $X = \{X_n; n \in \mathbf{Z}_1\}$, where $\mathbf{Z}_1 = \{1, 2, \ldots\}$, has the probability density $(2\pi 9)^{-1/2}\exp(-\frac{x^2}{18} + \frac{x}{3} - \frac{1}{2})$, $x \in I\!R$.

(i) Find the mean $\mu = EX_n$, and the variance σ^2 of $X_n; 1 \leq n < \infty$. (Hint: it is not necessary to use integration, just use the standard form of the Gaussian density.)

(ii) Use Chebychev's inequality to find an upper bound on the probability that X_n for any given $n, 1 \leq n < \infty$, takes a value greater than or equal to 3 units away from its mean.

Question 5.5

One is attempting to estimate the distribution function $F_X(x)$ of the random variable X at $x = 1$, i.e. to estimate $F_X(1)$, on the basis of n independent and identically distributed observations $\{X_1, X_2, ..., X_n\}$ of the random variable where each $X_i \sim F_X$. This is done by computing the relative frequency $r_A(n) = \frac{1}{n}\Sigma_{i=1}^n I_A(X_i)$, where A is the event $\{X \leq 1\}$.

Evaluate a Chebychev upper bound to

$$P(|r_A(n) - P(A)| \geq \epsilon.n^{-\frac{1}{2}})$$

in terms of $F_X(1)$.