ECSE 304-305B   Assignment 4   Winter 2008

Return by noon, Thursday, 9 October

Question 4.1 (SG p. 152)

Let X be a random point be selected from the interval $[0,3]$ (i.e. by the EPP, or, equivalently, such that X has a linearly increasing distribution function). What is the probability (expressed in terms of square roots if necessary) that

(i) $X^2 - 5X + 6 > 0$, and (ii) $X^2 - 5X + 6 \leq 1$?

Question 4.2

A student takes a certain type of test four times and the student's final score will be the maximum of the test score. Thus

$$\mathbf{X} = max\{\mathbf{X_1}, \mathbf{X_2}, \mathbf{X_3}, \mathbf{X_4}\}, \tag{1}$$

where $\mathbf{X_1}, \mathbf{X_2}, \mathbf{X_3}, \mathbf{X_4}$, are the four test scores and $\mathbf{X}$ is the final score. Assume the score takes values $x, 0 \leq x \leq 100$, and the results of the tests form an independent set of random variables, each with distribution function $F_{\mathbf{X_i}} = F(x) = P(\mathbf{X_i} \leq x), \quad 1 \leq i \leq 4$. Let the distribution function of $\mathbf{X}$ be written as

$$F_{\mathbf{X}}(x) = \{P(\mathbf{X} \leq x), \quad 0 \leq x \leq 100\}. \tag{2}$$

(a) Find which of the following gives $F_{\mathbf{X}}(\cdot)$ and give the derivation:

(i) $F^{1/4}(x^4)$,   (ii) $F^4(x)$,   (iii) $F(x^4)$

(b) If $F(x) = \{0, x < 0;\ \frac{x}{100},\ 0 \leq x \leq 100;\ 1, 100 \leq x\}$, and given that any score $\mathbf{X}_i$ is uniformly distributed over $[0, 100]$, find $F_{\mathbf{X}}(80) = Pr\{\mathbf{X} \leq 80\}$.

Question 4.3 For a constant parameter $a > 0$, a Rayleigh random variable X has PDF

$$f_X(x) = \begin{cases} a^2 x e^{-a^2x^2/2} & x > 0, \\ 0 & otherwise \end{cases}$$

What is the CDF of X?

Question 4.4 The cumulative distribution function of random variable U is

$$F_U(u) = \begin{cases} 0 & u < -5 \\ (u+5)/8 & -5 \le u < -3, \\ 1/4 & -3 \le u < 3, \\ 1/4 + 3(u-3)/8 & 3 \le u < 5, \\ 1 & u \ge 5. \end{cases}$$

(a) What is $E[U]$?

(b) What is $Var[U]$?

(c) What is $E[2^U]$?

Question 4.5 X is a continuous uniform (-4,4) random variable. Find the following:

(a) The PDF $f_X(x)$

(b) The CDF $F_X(x)$

(c) $E[X]$

(d) $E[X^5]$

(e) $E[e^X]$

4.6 The time until a small meteorite first lands anywhere in the Sahara desert is modeled as an exponential random variable with a mean (= the recipricol of the rate λ) of 10 days. The time is currently midnight. What is the probability (to two decimal places) that a meteorite first lands some time between 6 a.m. and 6 p.m. of the first day?